Extended Critical Values for a Simple Test for Cointegration

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Abstract

Leybourne and McCabe (1993) have extended the Kwiatkowski et al (1992) stationarity test to examine the null hypothesis of cointegration. The purpose of this note is to provide an extended set of critical values for use in applied research.

1. Introduction

Single equation tests for cointegration of the Engle-Granger type examine the null hypothesis of no cointegration. The tests are designed to determine whether the residuals from a cointegrating regression contain a unit root. Since classical hypothesis testing rejects the null only in the presence of very strong evidence against it, several authors have examined how changing the null hypothesis affects inference. For example, Kwiatkowski et al (1992) proposed a univariate test in which the null hypothesis is that a series is stationary (or trend-stationary). Used in conjunction with unit root tests, their test provides researchers with additional information on the temporal behaviour of a time series. Ideally, one would hope to find both tests suggestive of a unique description of a series as being stationary or not, but in practice, unit root and stationarity tests sometimes lead to conflicting inferences (see Baillie and Pecchenino (1991) and Kumar and Sephton (1994)).

Leybourne and McCabe (1993) extended the Kwiatkowski et al (1992) test to the residuals from a cointegration regression. In this framework, the null hypothesis is that the series are cointegrated. This allows a direct test of whether or not two or more series are cointegrated, since rejection of the null hypothesis in Engle-Granger type tests may be the result of specification error (ie. we may conclude that two or more series are cointegrated when in fact, they are not). While the potential for conflict between unit root and stationarity tests remains in the cointegration arena, the use of both types of tests will assist in the characterization and modelling of economic time series. The purpose of this note is to provide a more detailed set of critical values for the Leybourne and McCabe (1993) cointegration test, spanning a larger set of variables, and several sample sizes typically encountered in applied research.

The next section outlines the Leybourne and McCabe (1993) test, and is followed by a description of the Monte Carlo experiments used to estimate critical values of the tests. This leads to a table that can be used to determine the critical values of the test statistic for up to eleven variables and ten different sample sizes.

2. The Test Statistic

Consider a vector of variables all of which are known to be I(1), which follow the model

$$Y_t = \alpha_t + \mathbf{x}_t' \beta + \epsilon_t \qquad (1)$$

where we assume that

$$\epsilon_t \sim \text{I.N.}(0,\sigma^2), \quad \alpha_t = \alpha_{t-1} + \eta_t, \quad \alpha_0 = \alpha, \quad \eta_t \sim \text{I.N.}(0,\sigma^2 \sigma_\eta^2)$$

ϵ_t and η_t are mutually independent, and $\mathbf{x}_t'$ is a kx1 vector of I(1) variables.

The cointegration test takes the form of a test of the null hypothesis that $\sigma_\eta^2 = 0$ (against the one-sided alternative that it is positive). This follows from the components representation of the processes determining the dependent variable. If the intercept is constant, so that its variance is zero, then the series are linearly cointegrated (see Kwiatowski et al (1992) for the univariate case). That is, there is a linear combination that yields a stationary residual.

The form of the test statistic, denoted here by LMK, is given by equation (2):

$$LMK = T^{-2} s_\epsilon^{\ 2} e' V e$$

where $s_\epsilon^{\ 2}$ is a consistent estimator of σ_ϵ^2, e denotes the least squares residuals from the cointegrating regression over sample size T (under H_o), and V is a TxT matrix whose ijth element is equal to the minimum of i and j, i,j,=1,....,T.

Leybourne and McCabe (1993, p.100) note that the sum of squared residuals over T would be a consistent estimator of σ_ϵ^2 if the ϵ_t are I.I.D., but that in practice, this is unlikely to be the case. One might want to employ methods similar to Phillips and Perron (1988) to arrive at a consistent estimator of the error variance in practice, but for the purpose of estimating critical values of the LMK test statistic, I will employ the sum of squared residuals over T.

3. Simulated Test Statistics

The simulations involved 10,000 replications for each cointegrated system, spanning from two to eleven variables, and sample sizes 30, 50, 75, 100, 125, 150, 175, 200, 250, and 500 (Leybourne and McCabe examined up to six variables for a sample size of 500). On a Pentium 90 personal computer, the simulations for a sample size of 500 took approximately 40 hours, relegating a response surface approach to the estimation of critical values to future research.

For each sample size, at each iteration, ten series were constructed using NAG Mark 15 routines G05CBF, G05EAF, and G05EZF. Pseudo-random numbers were drawn from a multivariate normal distribution with zero mean and identity covariance matrix. I(1) series were then constructed using partial sums. Under the null hypothesis, the dependent variable was constructed for each system design (ie, from 2 to 11 variable systems), with $\alpha=\beta_1=,...,=\beta_k=1$, and ϵ_t constructed using routine G05FDF (as Leybourne and McCabe (1993,p.100) note, the choice of unit coefficients has no impact on the critical values). The cointegrating regression was implemented using routine G02DAF, and the test statistics were calculated and stored. After 10,000 replications, the test statistics were sorted, and the 90, 95, and 99 percent critical values were tabulated. Table 1 gives the simulated critical values by sample size and system design, as well as those reported by Leybourne and McCabe for T=500. My simulated critical values are surprisingly close to those reported by Leybourne and McCabe, given differences in computing and numerical routines. The only case in which there is a large difference in estimated critical values (.02 or more) is in the bivariate case for T=500, at the 99 percent level , and is most likely explain by differences in computing and numerical methods.

4. Conclusions

This paper provides small sample critical values for the Leybourne and McCabe (1993) test of cointegration for up to eleven variables. Used in conjunction with Engle-Granger type tests, this test helps ensure that appropriate inferences will be made with respect to whether or not series are cointegrated.

References

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Table 1: Critical Values of Cointegration Test by Sample Size and Number of Variables

Number of Variables	T=30	T=50	T=75	T=100	T=125	T=150	T=175	T=200	T=250	T=500	Lebourne McCabe
						90 Percent					
2	0.245	0.240	0.234	0.234	0.238	0.236	0.237	0.235	0.236	0.230	0.228
3	0.175	0.170	0.165	0.166	0.170	0.167	0.173	0.166	0.166	0.164	0.164
4	0.134	0.128	0.123	0.125	0.125	0.125	0.125	0.124	0.123	0.123	0.119
5	0.106	0.102	0.098	0.098	0.097	0.098	0.095	0.095	0.097	0.094	0.093
6	0.089	0.084	0.080	0.079	0.079	0.077	0.078	0.078	0.079	0.078	0.076
7	0.075	0.072	0.068	0.067	0.066	0.065	0.066	0.066	0.065	0.065	-
8	0.065	0.062	0.058	0.057	0.057	0.056	0.056	0.056	0.055	0.055	-
9	0.057	0.054	0.052	0.050	0.050	0.049	0.049	0.049	0.048	0.048	-
10	0.052	0.049	0.046	0.045	0.044	0.043	0.043	0.043	0.042	0.043	-
11	0.048	0.044	0.041	0.040	0.040	0.039	0.039	0.039	0.038	0.038	-
						95 Percent					
2	0.328	0.316	0.314	0.313	0.328	0.320	0.330	0.318	0.318	0.311	0.308
3	0.233	0.225	0.223	0.223	0.229	0.220	0.224	0.223	0.218	0.217	0.219
4	0.177	0.170	0.165	0.166	0.166	0.165	0.166	0.160	0.160	0.162	0.156
5	0.136	0.133	0.127	0.126	0.125	0.128	0.123	0.123	0.127	0.125	0.121
6	0.111	0.106	0.101	0.102	0.010	0.099	0.098	0.010	0.099	0.098	0.096
7	0.093	0.089	0.085	0.083	0.083	0.082	0.081	0.081	0.081	0.081	-
8	0.079	0.075	0.072	0.070	0.071	0.069	0.068	0.068	0.068	0.068	-
9	0.068	0.066	0.063	0.061	0.061	0.059	0.059	0.059	0.058	0.058	-
10	0.062	0.058	0.055	0.053	0.053	0.052	0.052	0.052	0.051	0.052	-
11	0.056	0.052	0.049	0.047	0.047	0.046	0.046	0.045	0.045	0.045	-
						99 Percent					
2	0.545	0.530	0.534	0.541	0.545	0.545	0.536	0.511	0.536	0.536	0.557
3	0.403	0.391	0.400	0.400	0.402	0.401	0.395	0.376	0.374	0.375	0.389
4	0.312	0.288	0.289	0.285	0.277	0.274	0.287	0.276	0.271	0.267	0.282
5	0.222	0.226	0.215	0.206	0.212	0.204	0.210	0.203	0.210	0.209	0.196
6	0.176	0.170	0.165	0.167	0.162	0.162	0.158	0.154	0.156	0.159	0.154
7	0.146	0.138	0.133	0.133	0.130	0.125	0.131	0.124	0.124	0.120	-
8	0.120	0.114	0.110	0.108	0.113	0.104	0.105	0.105	0.101	0.105	-
9	0.010	0.099	0.095	0.091	0.092	0.090	0.091	0.087	0.087	0.089	-
10	0.090	0.084	0.082	0.079	0.078	0.077	0.075	0.075	0.074	0.076	-
11	0.079	0.074	0.070	0.068	0.066	0.065	0.065	0.066	0.065	0.064	-