

**ASYMMETRIC FIRM ENTRY AND  
SOCIAL INEFFICIENCY**

by

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## **Asymmetric Firm Entry and Social Inefficiency**

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## 1. Introduction

Economists normally think of entry into an industry as increasing social efficiency. However a series of articles has established, when average costs decrease in firm output, that there can be too many firms. (See, among others, Perry (1984), Mankiw and Whinston (1986) and, for an empirical application, Berry and Waldfogel (1999)). This happens because while entry always increases industry output and reduces price, it also reduces firm output and thereby raises average cost. Entry is excessive because the latter effect, dubbed "business-stealing" by Mankiw and Whinston, can make entry more beneficial to the entrant than to society. These articles all assumed symmetric firms so as to more easily model entry as a change in firm numbers. This paper studies what happens when this simplifying assumption is relaxed so that firms can vary in size because of cost differences. It does so by building on a model introduced by Clarke and Davies (1982) to explain, among other things, industry concentration and firm size differences.

Of course dropping symmetry, while recognizing that firms do differ in size, means that entry is no longer as easily modeled. In symmetric models, entry occurs whenever price exceeds a common average cost. In contrast, when firms have different average costs, then entry occurs only if a potential entrant thinks its average cost is less than price. Moreover, the scale of entry, now an important entry characteristic, will depend on this firm's anticipated average cost relative to incumbents' average cost.

To analyze this more heterogeneous entry, we consider a specific model defined by Cournot equilibrium and linear market demand, where entry occurs when an outsider discovers

costs low enough to earn profits inside the industry. Our goal is to determine how entry and, in particular, scale of entry influences welfare, prices and concentration. Among our conclusions are that entry will reduce welfare unless the entrant's market share exceeds a critical level determined only by the number of post-entry firms. For example, if post-entry firms are 10, then welfare decreases if the entrant's share is below .0905. We also find, unlike the symmetric entry case, that entry can sometimes decrease industry average cost, increase industry profit and increase concentration as measured by the Herfindahl index (H). Moreover, if entry increases profits and concentration it must increase welfare. Finally, as with symmetric entry, entry always reduces price and increases consumer surplus. Our different results occur because, unlike the symmetric case, entry now redistributes output among firms of different efficiencies.<sup>1</sup>

In the following sections we introduce the model and discuss results.

## 2. Asymmetric Entry

We begin with an industry equilibrium as described by Clarke and Davies. The goal is to determine how the scale of the entrant affects price, costs and welfare. Our approach is to describe the equilibrium after entry and then compare this to the pre-entry equilibrium with  $N-1$  firms. Accordingly, after entry there are  $i=1, N$  firms, each with constant and different average costs  $c_i$ . Industry average cost is  $\sum s_i c_i$ , where  $s_i$  is firm  $i$ 's market share, and market demand is  $P = a - bQ$  with  $Q = \sum q_i$ . We also introduce the simple mean of firm average cost,  $u = \sum c_i / n$  and

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<sup>1</sup>This redistribution effect is also central in the equilibrium merger literature (see Farrell and Shapiro (1990)).

assume Cournot behavior.

Since each firm chooses  $q_i$  to maximize profits,  $\pi_i = P q_i - c_i q_i$ , the equilibrium condition for each firm is

$$P[1 - (q_i / Q)(1 / e)] = c_i \quad (1)$$

where  $e = -(P/Q)(dQ/dP)$  is the demand elasticity. Clarke and Davies show that equation (1) can be summed over the  $N$  firms and solved for  $P$ :

$$P = (Neu) / (Ne - 1) \quad (2)$$

Equation (1) can be re-expressed as  $(q_i / Q)$  or  $s_i = e [1 - c_i / P]$ , and after substituting for  $P$  from equation (2), this defines  $s_i$  as:

$$s_i = e(1 - c_i / u) + c_i / Nu \quad (3)$$

Next we also use equation (3) to explicitly define each firm's average cost:

$$c_i = (s_i - e) / [(1 / Nu) - e / u] \quad (4)$$

This leads directly to the Clarke-Davies expression for industry average cost:

$$AC = \frac{\sum_1^N s_i(s_i - e)}{[(1/Nu) - e/u]} = \frac{(H - e)}{[(1/Nu) - e/u]} \quad (5)$$

On the demand side, for linear demand,  $P = a - bQ$ , the elasticity is  $e = P/(a - P)$ , and substituting for  $P$  from equation (2) gives:

$$e = (Nu + a) / (a - u)N \quad (6)$$

and  $a = (Nu + Neu) / (Ne - 1) \quad (7)$

Finally, the slope of the demand curve is  $b = (a - P)/Q = (a - P)(P/R)$  where  $R$  is industry revenue. Substituting for intercept “a” from equation (7) and  $P$  from (2) expresses  $b$  as:

$$b = (N^2 u^2 e) / (Ne - 1)^2 R \quad (8)$$

To study entry, we take the post-entry equilibrium represented by the above equations, initialize the arithmetic mean of average costs at  $u = \sum c_i/n = 1$  and choose some arbitrary value for the elasticity. This defines, using equations (4), (7) and (8) firm  $i$ 's average cost and the intercept and slope of the demand curve, all of which are structural features that do not change because of entry. This starting point also defines, using equations (2) and (5), price and industry average cost

which, in contrast, are variables that do change because of entry.

One of the firms in this equilibrium is the entrant. By observing its share ( $s_e$ ), the entrant's average cost,  $c_e$  may be inferred from equation (4):

$$c_e = (s_e - e) / ((1 / Nu) - e / u) \quad (9)$$

The import of equation (9) is entry takes place because an outsider can produce at a cost,  $c_e$ , less than price. The lower this cost, the greater is the entrant's share. In fact, equation (9) is the key to linking the pre- and post-entry industry equilibrium, which is needed to determine how entry affects welfare. Specifically, because equation (9) tells us the entrant's cost, we can use this to determine the industry's cost before the entrant's appearance. With this information, the other pre-entry characteristics of the industry soon follow. Accordingly, based on equations (2) to (9) and using the subscript "b" to identify variables before entry, we can write:

$$u_b = (Nu - c_e) / (N - 1) = \frac{N - [(s_e - e) / ((1 / Nu) - e / u)]}{N - 1} \quad (10)$$

$$e_b = \frac{(N - 1)u_b + a}{(a - u_b)(N - 1)} = \frac{Ne + s_e}{N - s_e} \quad (11)$$

$$P_b = \frac{(N-1)e_b u_b}{(N-1)e_b - 1} = \frac{Ne + s_e}{Ne - 1} \quad (12)$$

$$Q_b = (a - P_b) / b = \frac{(Ne - 1)(N - s_e)R}{N^2 e} \quad (13)$$

$$s_{ib} = e_b(1 - c_i / u_b) + c_i / (N - 1)u_b = \frac{Ns_i + s_e}{N - s_e} \quad (14)$$

Equations (10) to (14) describe the industry before entry of scale  $s_e$ . Comparing equations (2) and (12) shows that prices fall due to entry by the proportion  $s_e/(Ne-1)$ . The business-stealing effect is evident from equation (14) since shares of incumbent firms before entry ( $s_{ib}$ ) exceed shares ( $s_i$ ) after entry. Equation (14) is crucial because we need to know how entry influences shares to determine how entry influences concentration and industry average cost. Accordingly we first use (14) to evaluate the Herfindahl index before entry:

$$H_b = \sum_{i=1}^{N-1} (s_{ib})^2 = \sum_{i=1}^{N-1} \frac{(N^2 s_i^2 + 2Ns_e s_i + s_e^2)}{(N - s_e)^2}$$

But since  $H - s_e^2 = \sum_{i=1}^{N-1} s_i^2$  and  $\sum_{i=1}^{N-1} s_i = 1 - s_e$ , therefore:

$$H_b = \frac{N^2(H - s_e^2) - (N + 1)s_e^2 + 2Ns_e}{(N - s_e)^2} \quad (15)$$

Equation (15) makes it possible to figure out exactly how entry or, for that matter, exit changes concentration by comparing  $H$  and  $H_b$ . (With symmetric entry,  $H$  and  $s_e = 1/N$  and (15) reduces to  $H_b = 1/(N-1)$ ).

Finally, with  $H_b$  defined, industry average cost before entry is an appropriately adjusted version of equation (5):

$$AC_b = \frac{(H_b - e_b)}{[(1/(N-1)u_b) - e_b/u_b]} = \frac{N[N(H - e) - (N + 1)s_e^2 + (e + 1)s_e]}{(N - s_e)(1 - Ne)} \quad (16)$$

Equation (16), together with equation (5), makes it possible to figure out how entry changes average cost. With average cost, price and output determined before and after entry, we can evaluate the welfare consequences of entry.

### 3. Consequences of Entry

In this section we see how entry changes industry profit ( $\pi$ ), consumer surplus (CS) and welfare ( $W=CS+\pi$ ). Post- entry profit, consumer surplus and welfare are respectively  $(P-AC)Q$ ,  $aQ-b(Q^2/2)-PQ$  and  $aQ-b(Q^2/2)-ACQ$  where  $Q=(a-P)/b$ . The expressions for  $a$ ,  $b$ ,  $P$ ,  $Q$  and  $AC$  are from equations (2) through (8). Analogously, pre-entry values for consumers surplus, profit and welfare are found using the derived expressions for  $P_b$ ,  $AC_b$ , and  $Q_b$ .

To more easily compare, we report profit, consumer surplus and welfare after entry of size  $s_e$ , and the change in each ( $d\pi$ ,  $dCS$ ,  $dW$ ) because of that entry. Thus,  $d\pi = \pi - \pi_b$  and so on.

$$\pi = (RH) / e, d\pi = \frac{(Rs_e)[s_e(N^2 + N + 1) - 2N]}{N^2 e} \quad (17)$$

$$CS = (R / 2e), dCS = \frac{Rs_e(2N - s_e)}{2N^2 e} \quad (18)$$

$$W = (R / 2e)(2H + 1), dW = \frac{Rs_e[s_e(2N^2 + 2N + 1) - 2N]}{2N^2 e} \quad (19)$$

Inspection of these equations shows immediately that entry always increases consumer surplus since  $dCS > 0$ . The effect on welfare and profit, however, depends on  $s_e$  relative to  $N$ .

Specifically:

$$d\pi \geq 0 \Rightarrow s_e \geq 2N / (N^2 + N + 1) \quad (20)$$

$$dW \geq 0 \Rightarrow s_e \geq 2N / (2N^2 + 2N + 1) \quad (21)$$

Equation (20) shows, if entry is large enough, that profits will increase even though entry always reduces price. This must be because sufficiently large entry will reduce average cost even more. Equation (21) also establishes that entry has to reach a threshold size before welfare can increase and this threshold is less than average size,  $1/N$ . Why does welfare fall if entry is small? The intuition is small entry increases industry average cost because it causes more efficient larger firms to reduce output (from (14)  $s_i < s_{ib}$ ). The increase in average costs, coupled with lower prices, means lower producer profits can more than offset higher consumer surplus. In contrast, larger entry means the entrant is, itself, one of the more efficient firms and average cost can fall.

Since the effect of entry on the distribution of output and average cost appears central, we look at  $dH = H - H_b$  and  $dAC = AC - AC_b$  and determine the scale of entry that would increase concentration and reduce average cost:

$$dH \geq 0 \Rightarrow s_e \geq \frac{2N(H+1)}{(N^2 + N + 1 + H)} \quad (22)$$

$$dAC \leq 0 \Rightarrow s_e \geq (H+1)/(N+1) \quad (23)$$

Entry is often seen as a concentration-reducing process, but, obviously, for large enough entry concentration will increase. However equation (22) implies the concentration-increasing size of the entrant can be fairly small when N is large. For example, if  $H=.25$  and  $N=100$  (an industry with numerous small firms), then according to (22) any  $s_e > .0247$  increases concentration. This must be because there are many firms smaller than the entrant who, in response to entry, reduce their output. The same argument explains equation (23) which says, for a given H, that a greater number of firms reduces the entry size required to lower average cost.

A clearer picture of the relationships is given in Figures 1 where  $d\pi$ ,  $dW$ ,  $dAC$  and  $dH$  are graphed against  $s_e$  for an industry where  $H=.25$ ,  $N=10$ ,  $e=1$  and  $R=1$ . The graph shows how the decreases in profit and welfare associated with small entry are connected to increases in average cost and decreases in H. In fact, the minimum  $s_e$  needed for a negative  $dAC$ ,  $(H+1)/(N+1)$ , is also the  $s_e$  where entry stops reducing H and begins to work in the other direction.<sup>2</sup> It can also be inferred from the graph that if entry increases concentration or profit, it

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<sup>2</sup>The derivative of  $dH$  with respect to  $s_e$  is zero when  $s_e=(H+1)/(N+1)=.1136$ .

also increases welfare.<sup>3</sup>

#### 4. Conclusion

As with the symmetric entry literature, we find that entry can reduce welfare. In our case the problem of excessive entry is not one of too many firms but of firms that are too small with costs that are too high. Small scale entry, because it directs output away from more efficient firms, can therefore be a welfare-reducing force. We also find, unlike the previous literature, that sufficiently large entry can increase profits and concentration. An irony, given that high concentration and profits, are sometimes offered as indicators of social inefficiency, is that entry that increases either always increases welfare. Another result is that we developed an expression, (see equation (15)), for measuring the effect of entry on the Herfindahl index that takes into account the output responses of incumbents.

At face value, the results imply entry could easily reduce welfare. For example, using equation (19), we can calculate the minimum  $s_e$  needed to ensure welfare does not fall. For  $N = (2, 5, 10, 25)$ , the corresponding critical  $s_e$  's are (.31, .16, .09, .038). These are quite large shares when placed against the observation (Dunne, Roberts and Samuelson (1988)) that small scale entry is common.

This paper has focused on entry in a setting consistent with the observation firms have different sizes. To accomplish this we used a Cournot model with linear demand that

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<sup>3</sup>Inspection of (21) and (22) shows the threshold  $s_e$  needed to increase H always increases W. Similarly from (20) and (21) if  $s_e$  increases profit it must increase welfare.

emphasized different technical abilities across firms. A question is the sensitivity of the conclusions to these assumptions. Two generalizations that can be inserted, neither of which appreciably changes the conclusions but add considerable complexity costs, are set-up costs and cooperative behavior indexed by positive conjectural variation elasticities.<sup>4</sup> Our conclusion is the model, although restrictive, can provide insight into how the size of entrant may influence, concentration, profit and welfare.

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<sup>4</sup>With set-up costs, the entrant's share needed to ensure welfare gains increases; it decreases with higher conjectural variation elasticities.

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**Figure 1.** Entry-induced changes in welfare, profit, concentration and costs

