# A Dynamic Model on Advertising Level and Product Quality

by

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# ABSTRACT

This report studies the relationship between advertising level and product quality where the product is a new non-durable experience good within a signaling game framework. The least cost separating equilibrium of the two period model used in Bagwell (2005) is analyzed without the restriction of advertising being dissipative with repeat business effect. The results are conditionally consistent with Bagwell (2005). Further, advertising level as a signal is assumed to be a random variable drawn from Normal and Beta distributions. Consumers update their prior beliefs about the quality of product by Bayes' rule after observing the advertising level. We show that pooling equilibria exist with the assumption of identical price and marginal costs for high quality and low quality firms. If asymmetric marginal costs are assumed, then the results are consistent with both Nelson (1974) and Schmalensee (1978).

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# 1. Introduction

Bagwell (2005) explores a potential relationship between advertising level for a new non-durable experience good[1] and product quality in a dynamic model. The assumptions in Bagwell (2005) serves as a starting point and a few different alternatives are examined. In most previous game theoretic literature on advertising level and the quality of product, it is common to assume that there are two discrete levels of quality, namely, the high-quality and the low-quality. In contrast, Linnemer (2011) examines a continuous range of quality options. These options are available to each firm at one point in time and under this static model each firm faces a continuous marginal cost curve and is uncertain about both quality and marginal cost.

Nelson (1970, 1974) points out that high quality firms may be technically more efficient and hence face a lower marginal cost curve. Under this assumption he derives the result that high quality firms can advertise more. He also mentioned that the advertising increased the initial sales for all firms but increased more repeat purchases for high-quality firms. In contrast to Nelson (1970, 1974), Schmalensee (1978) points out that low-quality product can be produced cheaper and hence the marginal cost of the high quality firm is higher than the low quality firm's marginal cost. Schmalensee (1978) also assumed the price of the high-quality firms are equal to the price of the low-quality firms. Under this assumption, he argued that the low quality firm has more room to advertise. As a result, the low quality firm can advertise more.

[1] Nelson (1970) defined an experience good as one for which quality can be only evaluated after purchase and consumption.

By considering the work of Nelson (1970, 1974) and Schmalensee (1978), Kihlstrom and Riordan (1984) offer a two-period model of advertising level and production choices made over time by a number of firms. The outcome of interaction among profit maximizing firms over time is called a repeat-business effect. With regard to repeated-business effects Kihlstrom and Riordan (1984) analyze the existence of equilibrium. In this two period model, the consumers in the first period are uncertain about the quality of what they are purchasing and the consumers in the second period will benefit from past experience as well as the general reputation of a good that has developed among them. Moreover, the high-quality firms and the low-quality firms have the same experience and reputation effect on their returns. Each firms is a competitive price taker and prices clear each market. There are two markets in both periods: one is the market for a high-quality good and the other is the market for a low-quality good. Kihlstrom and Riordan also assume that in the first period consumers use advertising level to predict the type of quality and in the second period consumers know the type of quality of each good. In the first period, new firms enter each market until the discounted profits reach zero. There is a minimum level of advertising cost required in order to enter the high-quality market in the first period. Advertising can be seen as an "entry fee"[2]. In this model, no matter whether the high-quality firms advertise or not, they establish their high-quality reputation ultimately. Under this result, repeat sales cannot be attributed to advertising and then the advertising can signal quality in first period if and only if the high-quality firm's investment only increases fixed costs and does not change marginal cost.

[2] The "entry fee" is first mentioned in Kihlstorm and Riordan (1984, P.432)

This model shows that an advertising equilibrium may exist with or without repeat-purchases. Repeat purchases are necessary for the existence of an advertising equilibrium only when marginal costs of high-quality firms are equal to those of the low-quality firms in the signaling models.

Kihlstorm and Riordan (1984) also slightly modify the information assumption mentioned above and consider advertising as a signal. They assume high-quality buyers only communicate with other high-quality buyers. With other conditions remaining the same, this new model demonstrates that if all but one high-quality firm advertises, then the non-advertising firm will not acquire a reputation for high-quality. In this case, Kihlstrom and Riordan (1984) point out that the advertising can lead to repeat purchase. The return on advertising for high-quality firms in the second model is higher than the return on advertising for high-quality firms in the first model. In that situation, even though low-quality firms face lower marginal cost, the advertising may still signal quality.

In the spirit of the models discussed above, Milgrom and Roberts (1986) consider a two-period model with dissipative advertising. The term ‘dissipative advertising’ implies advertising does not increase the demand directly, in other words, $D_A = 0$. This model involves repeat purchase, and it is interested a monopoly's equilibrium price and advertising choices and in the resulting consumer beliefs. Consumers are assumed to infer product quality based on advertising and price. This is quite different than the Kihstrom and Riordan (1984) model under which firms are price takers and hence price does not signal quality.

Milgrom and Roberts (1986) consider a newly introduced experience good and they focused on sequential equilibrium. There are two firms and each one choose a price (P) and an advertising level (A). Each firm's overall plan, or in other word strategy is the pair (P, A). The reason they assume that firms use the strategy (P, A) is because they assume that consumers make their initial purchase after they observe the price and advertising. Moreover, the advertising cost may increase in the second period. They also confirm that a pooling sequential equilibrium may exist if each firm's strategy is the same. Milgrom and Roberts (1986) also conclude and confirm that advertising and price may signal quality.

Finally, Bagwell (2005) studied the repeat-business effect with a dynamic signaling game model to analyze the least cost separating equilibrium when where is dissipative advertising. At the least cost separating equilibrium, customers can determine the quality of product by observing the firm's advertising level. The main idea of the Bagwell's repeat-business effect is that for the high-quality product, the return and the initial sale caused by advertising is greater than the low-quality product. This result is due to the satisfied customers who purchase the good more often. Bagwell (2005) also argued if the marginal cost of production for a high-quality product is higher than the marginal cost of production for a low-quality product, the high-quality producer will use dissipative advertising to signal quality in a least-cost separating equilibrium. Bagwell's model is explained in more detail in following section.

Moreover, Linnemer (2010) shows that in a static model with discrete two-dimensional types, the dissipative advertising is a necessary condition for the existence of separating equilibrium with uncertain and continuous quality and marginal cost.

In this report, first, the least cost separating equilibrium with repeat purchase effect is modeled in a way that is similar to Bagwell (2005). Second, the existence of a pooling equilibrium under which a low quality firm has an incentive to mimic a high quality firm is examined.

The next section presents the assumptions of Bagwell's model and the separating equilibrium in a two-period model with a discrete consumer belief is analyzed. At the separating equilibrium, consumers can distinguish between high quality good and low quality good by observing advertising levels which signal quality in a signaling game framework. Since there are just two levels of quality, consumer belief will either be 1 or 0 for high-quality and low-quality, respectively. Furthermore, the existence of equilibrium is examined when beliefs are continuously updated rather than discrete. Following this, other circumstances such as asymmetric marginal costs are examined. The following linear demand function is assumed throughout the report. The letter P stands for the price and b is the belief of consumers about the quality of the product after observing the advertising level A. The letters m and a are positive constants. $D(P, b) = a - m \frac{P}{b(A)}$. This demand function is increasing in b and decreasing in P.

## 2. Repeat Business Effect Model

The model will be similar to Bagwell's (2005) except that the demand function above will be used which does not guarantee dissipative advertising.

1. There are two types of quality t, $t \in (L, H)$, where L denotes the low-quality product, H denotes the high-quality product.
2. P is the price where $P \geq 0$.
3. A is the advertising level.
4. The marginal cost is c, c(H) for high-quality firm and c(L) for low-quality firm.
5. k is the unit cost of advertising, i.e. kA is the total cost for advertising.
6. b(A) is the consumer's updated belief about the quality of the product after observing the advertising level, b(A) $\in [0, 1]$, in other words, b is the conditional probability of being high-quality given the advertising level of A.
7. The demand function D (P, b) = a – m $\frac{P}{b(A)}$ where a, m are two positive constant parameters.
8. The two-period profit function is $V(P, A, b, t) = \Pi(P, A, b, t) + \delta\tilde{\pi}(P, b, t)$

   where δ is the discount factor which belongs to the interval $(0,1)$,

   $\Pi(P, A, b, t)$ is the profit function of the first period, and $\tilde{\pi}(P, b, t)$ is the reduced form profit function of the second period. Reduced form means that the function has fewer independent variables. Second period profit function does not depend on advertising level directly, It depends on A implicitly through the updated belief b since advertising occurs in the first period only.
9. The model is a monopoly model.

The type of game in this model is called the signaling game. The signaling game is a dynamic Bayesian game. There are two players, namely a sender and a receiver. The sender has a certain type t, which is given by nature. The sender observes his/her own type while the receiver does not know the type of the sender. Based on his/her knowledge of his/her own type, the sender chooses to send a message from a set of possible messages $M = \{m_1, m_2, m_3, ..., m_j\}$. The receiver observes the message but not the type of the sender. Then the receiver chooses an action from a set of feasible actions $A = \{a_1, a_2, a_3, ...., a_k\}$. The two players receive payoffs dependent on the sender's type, the message chosen by the sender, and the action chosen by the receiver [3]. The perfect Bayesian equilibrium requires that players' strategies be optimal given their beliefs and that their beliefs be consistent with the strategies of their opponents. There are two types of perfect Bayesian Nash Equilibrium in signaling games, namely pooling equilibrium and separating equilibrium. The pooling equilibrium is an equilibrium in which all types of senders send the same messages and the separating equilibrium is an equilibrium in which different types of senders send different messages.

### 2.1 Least-cost Separating Equilibrium

As discussed above, the separating equilibrium is an equilibrium in which different types of sender send different messages. The certain type of t is the quality of each firm, high-quality or low-quality. In this case, the separating equilibrium means the high-quality firms and low-quality firms have the different set of strategies (P, A) and different consumer believes, that is $(P_H, A_H)$

[3] The definition of signaling game is based on Gibbons, Robert (1992) and Osborne, M. J. & Rubenstein, A. (1994).

$\neq$ ($P_L$, $A_L$) and b ($A_H$) $\neq$ b ($A_L$). In the complete-information benchmark, the monopoly just has two selections, so the discrete variable of consumer belief is used. If they choose high-quality, the belief of high-quality will be one, and the belief of low-quality will be zero. According to the least-cost separating equilibrium, the best performance for the low-quality firm in the separating equilibrium with a complete-information selection is that it acts as a monopolist. Therefore, if the high-quality monopolist chooses some price-advertising pair (P,A) to make their profit less than or equal to what the low-quality monopolist gets [4] then it will prevent the low-quality monopolist from imitating high quality firm's signal. In other words, if it is very costly to mimic the high-quality firm's action, the low-quality monopolist will not mimic.

Moreover, Bagwell makes an important assumption about the reduced-form profit function for the second period in his two-period model due to a repeat-business effect. He assumes the reduced-form profit function will decrease if price increases and will increase if the consumer belief increases. More importantly, in order to show the repeat-business effect, Bagwell assumes the high-quality firm has a greater second period profit than the low-quality firms. From his point of view, when the price in the first period increases, some consumers will decide to stop buying and will leave. In this situation, Bagwell argues that there exists two different effects. First, he thinks that lost consumers may be more painful for the high-quality monopolist, since the high-quality monopolist would have a greater number of consumers who was satisfied in the first period and would return in the second period. Second, the high-quality monopolist may face less pain

[4] That is $V\ (P, A, b, L)\ \leq v_M(L)$

for the lost consumers, because the markup caused by increased price may be smaller, if the marginal cost increases with quality. The first consideration supports Bagwell's assumption, so Bagwell assume that if the c(H)≤ c(L), then $\tilde{\pi}_P(\mathrm{P,b,L}) > \tilde{\pi}_P(\mathrm{P,b,H})$. [5]

In the two-period model, for the first period, the profit function for either the low-quality firm or the high-quality firm is equal to the total revenue minus the total cost and cost of advertising. Therefore, the profit function is

$$\Pi(\mathrm{P,A,b,t}) = [\,\mathrm{a} - \mathrm{m}\,\frac{P}{b(A)}] \times (P - c) - kA \qquad (2.1)$$

Thus, because the consumers in the first-period have a choice to continue buying in the second period or not, so assume there exists a factor $\ell$, where $\ell$ is a random variable that shows the probability that a consumer finds the product satisfactory; in particular, it is the proportion of the first period consumer who remains in the second-period and $0 \leq \ell \leq 1$. Also, assuming $\ell_H > \ell_L$, which means the fraction of satisfied consumers in the high-quality monopolist is greater than in the low-quality monopolist.

Therefore, the reduced form profit function in the second period is

$$\tilde{\pi}(\mathrm{P,b,t}) = \ell\,\{[\,\mathrm{a} - \mathrm{m}\,\frac{P}{b(A)}] \times (P - c)\} \qquad (2.2)$$

Since there is no advertising in the second period, the term "$kA$" does not exist in equation 2.2. The number of consumers in the second period will be less than or equal to the number of the first period's consumers. There will not be any new consumers in the second period. Therefore,

[5] Please see Bagwell (2005)'s paper for more details on page 91

according to Bagwell's (2005) two-period profit function, the new two-period profit function is:

$$\mathrm{V(P, A, b, t)} = [\,\mathrm{a} - \mathrm{m}\ \frac{P}{b(A)}] \times (P-c) - kA + \delta\ell\,\{[\,\mathrm{a} - \mathrm{m}\ \frac{P}{b(A)}] \times (P-c)\} \qquad (2.3)$$

As mentioned before, in the least-cost separating equilibrium, the consumer beliefs about the high-quality monopolist and low-quality monopolist are different where b ($A_H$) = 1. And if the low-quality monopolist wants to mimic the high-quality monopolist's action, he/she will choose the same advertising level as the high-quality monopolist. Under these circumstances, the customer belief b of profit function is equal to 1. By setting the customer belief b =1, the updated two-period profit function is:

$$\mathrm{V(P, A, 1, t)} = [\,\mathrm{a} - \mathrm{m}\ P] \times (P-c) - kA + \delta\ell\,\{[\,\mathrm{a} - \mathrm{m}\ P] \times (P-c)\}$$

where $\tilde{\pi}(\mathrm{P, b, t}) = \ell\,\{[\,\mathrm{a} - \mathrm{m}\ P] \times (P-c)\}$ and $\tilde{\pi}$ is decreasing in *P*.

From Bagwell (2005), the least cost separating equilibrium equation is:

$$\max_{P,A} V\,(P, A, b, H) \text{ subject to } V\,(P, A, b, L) \le v_M(L) \qquad (2.4)$$

where $v_M(L)$ is the discounted two-period profit that the low-quality monopolist earns in a separating equilibrium. Moreover, according to the definition of separating equilibrium, the price here is the price of the high-quality monopolist.

In addition, the derivative of demand respect to advertising level A is

$$D_A = D_b \times b_A = \mathrm{m}\ \frac{P}{b(A)^2}\ b_A$$

Since the customer belief b =1, then $D_A = \mathrm{mP}b_A$. The sign of the derivative of demand depends on the value of derivative of consumer belief and the advertising level will not be dissipative advertising if $b_A \neq 0$. If $b_A \neq 0$, then $D_A \neq 0$.

The Lagrangian function derived from （2.4） is

$$\mathcal{L}(\mathrm{P},\mathrm{A},\lambda) = V\,(\mathrm{P},\mathrm{A},1,\mathrm{H}) + \lambda\,[v_M(L) - V\,(P,A,1,L)]$$

Then $$\frac{d\mathcal{L}}{dA} = \frac{\partial\,V\,(P,A,1,H)}{\partial A} - \lambda\,\frac{\partial V\,(P,A,1,L)}{\partial A} = 0$$

$$V_A\,(P,A,1,H) = \lambda V_A\,(P,A,1,L)$$

The F.O.C of $V$ respect to A is $$V_A\,(P,A,1,H) = -k$$

$$V_A\,(P,A,1,L) = -k$$

Hence $$\lambda = \frac{V_A\,(P,A,1,H)}{V_A\,(P,A,1,L)} = 1$$

Also $$\frac{d\mathcal{L}}{dP} = \frac{\partial\,V\,(P,A,1,H)}{\partial P} - \lambda\,\frac{\partial V\,(P,A,1,L)}{\partial P} = 0$$

Therefore $$V_P\,(P,A,1,H) = \lambda V_P\,(P,A,1,L) = V_P\,(P,A,1,L) \qquad \text{since } \lambda = 1$$

Then $$\Pi_P(\mathrm{P},\mathrm{A},1,\mathrm{H}) + \delta\tilde{\pi}_P(\mathrm{P},1,\mathrm{H}) = \Pi_P(\mathrm{P},\mathrm{A},1,\mathrm{L}) + \delta\tilde{\pi}_P(\mathrm{P},1,\mathrm{L})$$

$$\Pi_P(\mathrm{P},\mathrm{A},1,\mathrm{H}) - \Pi_P(\mathrm{P},\mathrm{A},1,\mathrm{L}) = \delta[\,\tilde{\pi}_P(\mathrm{P},1,\mathrm{L}) - \tilde{\pi}_P(\mathrm{P},1,\mathrm{H})]$$

The F.O.C of $\Pi$ respect to P is $$\Pi_P(\mathrm{P},\mathrm{A},1,\mathrm{H}) = a - 2Pm + mc(H)$$

$$\Pi_P(\mathrm{P},\mathrm{A},1,\mathrm{L}) = a - 2Pm + mc(L)$$

The F.O.C of $\tilde{\pi}$ respect to P is $$\tilde{\pi}_P(\mathrm{P},1,\mathrm{H}) = \ell_H\,\{a - 2Pm + mc(H)\}$$

$$\tilde{\pi}_P(\mathrm{P},1,\mathrm{L}) = \ell_L\,\{a - 2Pm + mc(L)\}$$

Thus, $$m[c(H) - c(L)] = \delta\{\ell_L[a - 2Pm + mc(L)] - \ell_H\,[a - 2Pm + mc(H)]\} \qquad (2.5)$$

## 2.2 Asymmetric Marginal Costs

Two cases of asymmetric marginal costs will be examined.

i. If $c(H) > c(L)$ , the left side of equation 2.5 will be positive. For the right side of equation 2.5, assuming that $\tilde{\pi}$ is decreasing in $P$, then $\tilde{\pi}_P < 0$. Both $\tilde{\pi}_P(\mathrm{P}, 1, \mathrm{H})$ and $\tilde{\pi}_P(\mathrm{P}, 1, \mathrm{L})$ are negative values. Since $\ell_H > \ell_L$ and both $\ell_H, \ell_L$ bigger than zero. Then $[a - 2Pm + mc(H)]$ and $[a - 2Pm + mc(L)]$ are also negative values. Thus, if $c(H) > c(L)$ then $[a - 2Pm + mc(H)] > [a - 2Pm + mc(L)]$ , so that the absolute value of $[a - 2Pm + mc(H)]$ is less than the absolute value of $[a - 2Pm + mc(L)]$. The sign of the right-hand side depends on the value of $\ell_H$ and $\ell_L$. Hence, it may be possible for the high-quality monopolist to signal its quality with a non-dissipative advertising as long as they choose price P which satisfies equation 2.5, conditional on the values $\ell_H$ and $\ell_L$ guarantee the positivity of the right hand side.

ii. If $c(H) < c(L)$, then the left side of equation 2.5 is negative. For the sign of the right side of equation 2.5, the inequality equation $a - 2Pm + mc(H) < a - 2Pm + mc(L)$ can be examined. First, both of them are negative values, so the absolute value of $a - 2Pm + mc(H)$ is bigger than the absolute value of $a - 2Pm + mc(L)$. Since $\ell_H > \ell_L$, then $\ell_H [a - 2Pm + mc(H)] < \ell_L[a - 2Pm + mc(L)]$, so that the sign of the right side should be positive. Hence, the equation cannot be solved.

The high-quality monopolist can only signal its quality with a non-dissipative advertising when its marginal cost is higher than the marginal cost of low-quality monopolist, given that the values of $\ell_H$ and $\ell_L$ satisfy the equation 2.5. In other words, when the non-dissipative advertising is used instead of the dissipative advertising, the results are conditionally consistent with Bagwell (2005).

# 3. <u>Continuous Consumer Belief</u>

It will be assumed that advertising level A is a random variable distributed by normal distribution first and later by beta distribution for robustness. After observing the advertising level A, the consumers update their beliefs b about the quality of the product by using Bayes' rule, so that b(A) is the conditional probability of being high quality given the advertising level A. For simplicity, it will be assumed that $\ell_H = \ell_L$ and $\mathrm{P}_H = \mathrm{P}_L$, unless stated otherwise.

## 3.1 Normal Distribution

The normal distribution probability function is $\mathrm{f}(\mathrm{x}, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{(\mathrm{x}-\mu)^2}{2\sigma^2}}$, where x is the random variable, $\mu$ is the mean value of x and $\sigma$ is the standard deviation of x. The customers' updated belief b depends on the advertising level A. As A is a random variable distributed by normal distribution, the advertising levels of high-quality firms distributed normally by $f_H(A)$ and the advertising levels of low-quality firms distributed normally by $f_L(A)$ can be written as,

$$\begin{cases} f_H(A) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2\sigma^2}(A-\mu_H)^2\right] \\ f_L(A) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2\sigma^2}(A-\mu_L)^2\right] \end{cases}$$

Given that, $Prob(H) = \gamma$, $Prob(L) = 1 - \gamma$

which means the prior probability of being high-quality is $\gamma$ and the prior probability of being low-quality is $1 - \gamma$

Observing the advertising level A, consumers update their prior belief by Bayes' Rule,

$$b(A) = Prob(H \mid A) = \frac{Prob(H \cap A)}{Prob(H \cap A) + Prob(L \cap A)}$$
$$= \frac{Prob(A \mid H) \times Prob(H)}{Prob(A \mid H) \times Prob(H) + Prob(A \mid L) \times Prob(L)}$$

where H is the high-quality product, and L is the low-quality product,

The $Prob(A \mid H)$ denotes the probability of advertising being equal to A given that the firm is a high-quality firm, and $Prob(A \mid L)$ denotes the probability of advertising being equal to A given that the firm is a low-quality firm.

Because $$\begin{cases} Prob(A \mid H) = f_H(A) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp[-\frac{1}{2\sigma^2}(A - \mu_H)^2] \\ Prob(A \mid L) = f_L(A) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp[-\frac{1}{2\sigma^2}(A - \mu_L)^2] \end{cases}$$

Then $$b(A) = \frac{\gamma}{\gamma + (1-\gamma) \cdot \exp[\frac{1}{2\sigma^2}(\mu_H^2 - \mu_L^2) + (\frac{\mu_L - \mu_H}{\sigma^2})A]} \quad (3.1)$$ [6]

Since the value of the exponential function is always positive, the denominator of equation 3.1 will be positive, and thus b(A) will always be positive. Taking b(A) into the demand function D (P, b) = a – m $\frac{P}{b}$ , the relationship between b and D will be positive. If b increases, then D

[6] This result is obtained by Zhonghui Wang's report.

increases and if b decreases, then D decreases.

If $\mu_H > \mu_L$, then $(\frac{\mu_L-\mu_H}{\sigma^2})$ will be negative. As advertising level A increases, the exponential function exp(·) will decrease and the denominator of equation 3.1 will also decrease. Therefore, b(A) increases, thus the demand D increases. As a result, if $\mu_H > \mu_L$, b is increasing in A and then D is increasing in A. In a word, the advertising level A is the demand enhancing adverting when $\mu_H > \mu_L$.

If $\mu_H < \mu_L$, then $\left(\frac{\mu_L-\mu_H}{\sigma^2}\right)$ will be positive. As advertising level A increases, the exponential function exp(·) will increase and this leads to a decrease in b(A). Hence, D also decreases.

By substituting b(A) in to equation 2.1 and 2.2,

$$\Pi(\mathrm{P,A,b,t}) = (P-c)\{\mathrm{a}-\mathrm{m}\,P\frac{\gamma+(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}\}-kA$$

$$\tilde{\pi}(\mathrm{P,b,t}) = \ell\,\{(P-c)[\mathrm{a}-\mathrm{m}\,P\frac{\gamma+(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}]\}$$

So that
$$\mathrm{V(P,A,b,t)} = (P-c)\left\{\mathrm{a}-\mathrm{m}\,P\frac{\gamma+(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}\right\}-kA + \ell\left\{(P-c)\left[\mathrm{a}-\mathrm{m}\,P\frac{\gamma+(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}\right]\right\}$$

The following is the first order derivative of V with respect to A,

$$V_A\,(P,A,b,t) = \frac{\partial V}{\partial A} = -(P-c)\mathrm{m}P(\frac{\mu_L-\mu_H}{\sigma^2})\frac{(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}$$

$$-k-\ell(P-c)\mathrm{m}P(\frac{\mu_L-\mu_H}{\sigma^2})\frac{(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}$$

$$=(1+\ell)(P-c)\mathrm{m}P\left(\frac{\mu_H-\mu_L}{\sigma^2}\right)\frac{(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}-k \qquad (3.2)$$

The second order derivative of V with respect to A, $V_{AA}$ is equal to

$$\frac{\partial}{\partial A}\left(\frac{\partial V}{\partial A}\right) = -(1+\ell)(P-c)\mathrm{m}P(\frac{\mu_H-\mu_L}{\sigma^2})\text{^}2\,\frac{(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma} < 0 \qquad (3.3)$$

Hence, there exists a maximum V(·)

### *3.1.1 Identical marginal cost*

Assuming the marginal cost of high-quality firms is equal to the marginal cost of low-quality firms, we will consider the following cases:

i. If $\mu_H > \mu_L$ , then $(\frac{\mu_H-\mu_L}{\sigma^2})$ will be positive and $V_A$ will be the difference between a positive value and $k$. Since second order derivative is negative, the profit can attain its maximum value.

Now, an optimal value of A can be examined by setting equation 3.2 equal to zero.

Setting $V_A\,(P,A,b,t) = (1+\ell)(P-c)\mathrm{m}P\left(\frac{\mu_H-\mu_L}{\sigma^2}\right)\frac{(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma}-k=0$

$$\frac{(1-\gamma)\cdot\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A]}{\gamma} = \frac{k}{(1+\ell)(P-c)\mathrm{m}P\left(\frac{\mu_H-\mu_L}{\sigma^2}\right)}$$

Then $\exp[\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A] = \frac{k}{(1+\ell)(P-c)\mathrm{m}P\left(\frac{\mu_H-\mu_L}{\sigma^2}\right)}(\frac{\gamma}{1-\gamma})$

$$\frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)+(\frac{\mu_L-\mu_H}{\sigma^2})A = \ln k - \ln\{(1+\ell)(P-c)\mathrm{m}P\left(\frac{\mu_H-\mu_L}{\sigma^2}\right)(\frac{\gamma}{1-\gamma})\} \quad .$$

Therefore $A^* = \frac{\ln k - \ln\{(1+\ell)(P-c)\mathrm{m}P\left(\frac{\mu_H-\mu_L}{\sigma^2}\right)(\frac{\gamma}{1-\gamma})\} - \frac{1}{2\sigma^2}(\mu_H^2-\mu_L^2)}{(\frac{\mu_L-\mu_H}{\sigma^2})}$ (3.4)

where $A^*$ is the optimal value.

If $\mu_H > \mu_L$, then $\left(\frac{\mu_H-\mu_L}{\sigma^2}\right) > 0, \left(\frac{\mu_L-\mu_H}{\sigma^2}\right) < 0$ and $\ln\{(1+\ell)(P-c)\,\mathrm{m}P\left(\frac{\mu_H-\mu_L}{\sigma^2}\right)(\frac{\gamma}{1-\gamma})\} > 0$. Since the numerator of equation 3.4 is negative, then, whole equation will be positive. Since A is the advertising level and will always be positive, so equation 3.4 can hold. Therefore, the optimal advertising $A^*$ exists. Hence, if the marginal cost of high-quality firms is equal to the marginal cost of low-quality firm, both firms will get the same optimal value of advertising level; therefore the pooling equilibrium exists, so that the low-quality monopolists will have incentive to mimic the high-quality monopolists.

In other words, given that both types have equal marginal costs and equal prices, if the mean advertising level of high quality firms in similar industries is higher than the mean of low quality firms' advertising level, then the optimal advertising levels are equal for both types. However, one must note that there exists a possibility of distorted price for the high quality firm. If high quality firm can distort the price, then it is possible to get a different result.

ii. If $\mu_H < \mu_L$ , then $(\frac{\mu_H-\mu_L}{\sigma^2})$ will be negative, and $V_A$ will have a negative value. Hence, V is a decreasing function in A. In other words, if the advertising level increases, the profit will decrease. In this case, the maximum profit value would be attained when A = 0. Hence, for both firms the optimal advertising level becomes A =

0. This is also a pooling equilibrium.

### *3.1.2 Asymmetric marginal costs*

Both of those two cases assume that the marginal cost of high-quality firms is equal to the marginal cost of low-quality firms. What if they are not equal?

Suppose c(H) > c(L), then $P - c(H) < P - c(L)$. Given that $\mu_H > \mu_L$, $\ln\{(1+\ell)(P - c(H))\mathrm{m}P\left(\frac{\mu_H - \mu_L}{\sigma^2}\right)(\frac{\gamma}{1-\gamma})\}$ is smaller than $\ln\{(1+\ell)(P - c(L))\mathrm{m}P\left(\frac{\mu_H - \mu_L}{\sigma^2}\right)(\frac{\gamma}{1-\gamma})\}$, and therefore $A_H^* < A_L^*$. The optimal advertising level of the high-quality monopolist is smaller than the optimal advertising level of the low-quality monopolist, which shows that the low-quality firm advertises more. This result is consistent with Schmalensee (1978).

On the other hand, if $c(H) < c(L)$, then $P - c(H) > P - c(L)$. So $\ln\{(1+\ell)(P - c(H))\mathrm{m}P\left(\frac{\mu_H - \mu_L}{\sigma^2}\right)(\frac{\gamma}{1-\gamma})\}$ is bigger than $\ln\{(1+\ell)(P - c(L))\mathrm{m}P\left(\frac{\mu_H - \mu_L}{\sigma^2}\right)(\frac{\gamma}{1-\gamma})\}$, and therefore $A_H^* > A_L^*$. The optimal advertising level of the high-quality firm is higher than the low – quality firm's advertising level. This result is consistent with Nelson (1974).

## 3.2 Beta Distribution

Now, it will be assumed that the advertising levels for two types are drawn from beta distributions symmetrically. The beta distribution probability function is $\mathrm{f}(\mathrm{x}, \alpha, \beta) = \frac{1}{B(\alpha,\beta)} \mathrm{x}^{\alpha-1}(1-\mathrm{x})^{\beta-1}$ where B is the beta function that normalizes the constant to ensure that the total probability integrates to 1 where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. The probability functions for high quality firms and low quality firms can be written respectively as below:

$$\begin{cases} f_H(A) = \frac{A^{\alpha_H - 1}(1-A)^{\beta_H - 1}}{B(\alpha_H, \beta_H)} \\ f_L(A) = \frac{A^{\alpha_L - 1}(1-A)^{\beta_L - 1}}{B(\alpha_L, \beta_L)} \end{cases}.$$

The prior probability of being high-quality is $\gamma$ and the prior probability of being low-quality is $1-\gamma$:

$Prob(H) = \gamma$, $Prob(L) = 1-\gamma$.

Observing the advertising level A, consumers update their prior belief by Bayes' Rule,

$$b(A) = Prob(H \mid A) = \frac{Prob(H \cap A)}{Prob(H \cap A) + Prob(L \cap A)}$$

$$= \frac{Prob(A \mid H) \times Prob(H)}{Prob(A \mid H) \times Prob(H) + Prob(A \mid L) \times Prob(L)}$$

where H is the high-quality product, and L is the low-quality product

The $Prob(A \mid H)$ denotes the probability of advertising level being equal to A given that the firm is a high-quality firm, and $Prob(A \mid L)$ denotes the probability of advertising level being equal to A given that the firm is a low-quality firm.

Because $\begin{cases} Prob(A \mid H) = f_H(A) = \frac{A^{\alpha_H - 1}(1-A)^{\beta_H - 1}}{B(\alpha_H, \beta_H)} \\ Prob(A \mid L) = f_L(A) = \frac{A^{\alpha_L - 1}(1-A)^{\beta_L - 1}}{B(\alpha_L, \beta_L)} \end{cases}$

From Bayes' Rule, $b(A) = \frac{\gamma}{\gamma + \frac{B(\alpha_H, \beta_H)}{B(\alpha_L, \beta_L)} \cdot A^{\alpha_L - \alpha_H}(1-A)^{\beta_L - \beta_H} \cdot (1-\gamma)}$ [7]

[7] This result is also taken from Zhonghui Wang's report.

Since $\begin{cases} B(\alpha_H, \beta_H) = \frac{\Gamma(\alpha_H)\Gamma(\beta_H)}{\Gamma(\alpha_H + \beta_H)} \\ B(\alpha_L, \beta_L) = \frac{\Gamma(\alpha_L)\Gamma(\beta_L)}{\Gamma(\alpha_L + \beta_L)} \end{cases}$ , then $\frac{B(\alpha_H,\beta_H)}{B(\alpha_L,\beta_L)} = \frac{\Gamma(\alpha_H)\Gamma(\beta_H)}{\Gamma(\alpha_H + \beta_H)} / \frac{\Gamma(\alpha_L)\Gamma(\beta_L)}{\Gamma(\alpha_L + \beta_L)}$

Assuming that the distribution of high-quality firms is being symmetric to the distribution of low-quality firms, there exists two cases:

1. $f_H(A)$ is increasing and $f_L(A)$ is decreasing symmetrically[8]
2. $f_H(A)$ is decreasing and $f_L(A)$ is increasing symmetrically.[9]

For both of those cases because the $f_H(A)$ and $f_L(A)$ are symmetric, then

1. $\alpha_H = \beta_L$ , the alpha value of high-quality firms equal to the beta value of low-quality firms
2. $\alpha_L = \beta_H$ , the alpha value of low-quality firms equal to the beta value of high-quality firm

Replace all the $\beta_L$ to $\alpha_H$, and replace all the $\beta_H$ to $\alpha_L$

As a result, $\frac{B(\alpha_H,\beta_H)}{B(\alpha_L,\beta_L)} = \frac{\Gamma(\alpha_H)\Gamma(\beta_H)}{\Gamma(\alpha_H + \beta_H)} / \frac{\Gamma(\alpha_L)\Gamma(\beta_L)}{\Gamma(\alpha_L + \beta_L)} = \frac{\Gamma(\alpha_H)\Gamma(\alpha_L)}{\Gamma(\alpha_H + \alpha_L)} / \frac{\Gamma(\alpha_L)\Gamma(\alpha_H)}{\Gamma(\alpha_L + \alpha_H)} = 1$

Therefore, $b(A) = \frac{\gamma}{\gamma + A^{\alpha_L - \alpha_H}(1-A)^{\beta_L - \beta_H} \cdot (1-\gamma)} = \frac{\gamma}{\gamma + A^{\alpha_L - \alpha_H}(1-A)^{\alpha_H - \alpha_L} \cdot (1-\gamma)}$ (3.5)

In addition, in the beta distribution, the mean values of both type of firms are,

$$\begin{cases} \mu_H = \frac{\alpha_H}{\alpha_H + \beta_H} \\ \mu_L = \frac{\alpha_L}{\alpha_L + \beta_L} \end{cases}$$

8 The graph1 is in Appendix

9 The graph2 is in Appendix

Since A is a positive value, the power function of A will always be positive. Therefore, the value of b(A) is also positive. Taking b(A) into the demand function D (P, b) = a – m $\frac{P}{b}$, it is clearly to see that the relationship between b and D is positive. As b increases, D will also increases and vice versa.

If $\mu_H > \mu_L$, then $\frac{\alpha_H}{\alpha_H+\beta_H} > \frac{\alpha_L}{\alpha_L+\beta_L}$ . Thus $\frac{\alpha_H}{\alpha_H+\alpha_L} > \frac{\alpha_L}{\alpha_L+\alpha_H}$ if and only if $\alpha_H > \alpha_L$. If $\alpha_H > \alpha_L$, then $A^{\alpha_L-\alpha_H}$ will decrease as advertising level A increases, and $(1-A)^{\alpha_H-\alpha_L}$ will decrease. Therefore, the denominator of equation 3.5 will decrease. Thus, b(A) increases as A increases, and D increases as A increases. Therefore, the advertising level A is the demand enhancing adverting when $\mu_H > \mu_L$.

If $\mu_H < \mu_L$, then $\frac{\alpha_H}{\alpha_H+\beta_H} < \frac{\alpha_L}{\alpha_L+\beta_L}$, thus $\frac{\alpha_H}{\alpha_H+\alpha_L} < \frac{\alpha_L}{\alpha_L+\alpha_H}$ if and only if $\alpha_H < \alpha_L$. If $\alpha_H < \alpha_L$, then $A^{\alpha_L-\alpha_H}$ will increase as advertising level A increases, and $(1-A)^{\alpha_H-\alpha_L}$ will increase. All of them will lead the denominator of equation 3.5 to increases. Thus, b(A) will decrease when A increases, so that D will also decreases when A increases.

If b(A) is substituted in to equation 2.1 and 2.2, we have

$$\Pi(\mathrm{P,A,b,t}) = (P-c)\{\text{ a – m } P\frac{\gamma + A^{\alpha_L-\alpha_H}(1-A)^{\alpha_H-\alpha_L}\cdot(1-\gamma)}{\gamma}\} - kA$$

$$\widetilde{\pi}(\mathrm{P,b,t}) = \ell\,\{(P-c)[\text{ a – m } P\frac{\gamma+A^{\alpha_L-\alpha_H}(1-A)^{\alpha_H-\alpha_L}\cdot(1-\gamma)}{\gamma}]\}$$

So that $V(P,A,b,t) = (P-c)\left\{ a - mP\frac{\gamma + A^{\alpha_L-\alpha_H}(1-A)^{\alpha_H-\alpha_L}\cdot(1-\gamma)}{\gamma}\right\} - kA$

$$+ \ell\left\{(P-c)\left[a - mP\frac{\gamma + A^{\alpha_L-\alpha_H}(1-A)^{\alpha_H-\alpha_L}\cdot(1-\gamma)}{\gamma}\right]\right\}$$

$$= (P-c)\left\{ a - mP\frac{\gamma + \left(\frac{1}{A}-1\right)^{\alpha_H-\alpha_L}\cdot(1-\gamma)}{\gamma}\right\} - kA$$

$$+ \ell\left\{(P-c)\left\{ a - mP\frac{\gamma + \left(\frac{1}{A}-1\right)^{\alpha_H-\alpha_L}\cdot(1-\gamma)}{\gamma}\right\}\right\}.$$

The first order derivative of V with respect to A is below.

$$V_A\,(P,A,b,t) = \frac{\partial V}{\partial A} = (P-c)\mathrm{m}P\frac{(1-\gamma)(\alpha_H-\alpha_L)}{\gamma}\left(\frac{1}{A}-1\right)^{\alpha_H-\alpha_L-1}\left(\frac{1}{A^2}\right) - k$$

$$+\ell(P-c)\mathrm{m}P\frac{(1-\gamma)(\alpha_H-\alpha_L)}{\gamma}\left(\frac{1}{A}-1\right)^{\alpha_H-\alpha_L-1}\left(\frac{1}{A^2}\right)$$

$$= (1+\ell)(P-c)\mathrm{m}P\frac{(1-\gamma)(\alpha_H-\alpha_L)}{\gamma}\left(\frac{1}{A}-1\right)^{\alpha_H-\alpha_L-1}\left(\frac{1}{A^2}\right) - k$$

$$= (1+\ell)(P-c)\mathrm{m}P\frac{(1-\gamma)}{\gamma}(\alpha_H-\alpha_L)\frac{(1-A)^{\alpha_H-\alpha_L-1}}{A^{\alpha_H-\alpha_L+1}} - k \qquad (3.6)$$

The second order derivative of V with respect to A is

$$V_{AA} = \frac{\partial}{\partial A}\left(\frac{\partial V}{\partial A}\right)$$

$$= (1+\ell)(P-c)\mathrm{m}P\frac{(1-\gamma)}{\gamma}(\alpha_H-\alpha_L)\times\left\{\frac{(\alpha_H-\alpha_L-1)(1-A)^{\alpha_H-\alpha_L-2}}{A^{\alpha_H-\alpha_L+1}} - \frac{(\alpha_H-\alpha_L+1)(1-A)^{\alpha_H-\alpha_L-1}}{A^{\alpha_H-\alpha_L+2}}\right\}.$$

### *3.2.1 Identical marginal cost*

It will be first assumed that the marginal cost of high-quality firm is equal to the marginal cost of low-quality firm.

i. If $\mu_H > \mu_L$, then $\alpha_H > \alpha_L$. Therefore, $\frac{(1-A)^{\alpha_H-\alpha_L-1}}{A^{\alpha_H-\alpha_L+1}}$ in equation 3.6 will be positive, and this leads to $V_A$ is the difference between a positive number and $k$. The factor $\{\frac{(\alpha_H-\alpha_L-1)(1-A)^{\alpha_H-\alpha_L-2}}{A^{\alpha_H-\alpha_L+1}} - \frac{(\alpha_H-\alpha_L+1)(1-A)^{\alpha_H-\alpha_L-1}}{A^{\alpha_H-\alpha_L+2}}\}$ will be negative and this leads $V_{AA}$ be negative. In this situation, there exists an optimal value A to maximize the profit.

Setting $V_A\,(P,A,b,t) = (1+\ell)(P-c)\mathrm{m}P\,\frac{(1-\gamma)}{\gamma}\,(\alpha_H-\alpha_L)\frac{(1-A)^{\alpha_H-\alpha_L-1}}{A^{\alpha_H-\alpha_L+1}} - k\ = 0$

Then $\frac{(1-A^*)^{\alpha_H-\alpha_L-1}}{A^{*\alpha_H-\alpha_L+1}} = \frac{k}{(1+\ell)(P-c)\mathrm{m}P\frac{(1-\gamma)}{\gamma}(\alpha_H-\alpha_L)}$ (3.7)

The optimal value of A should satisfy the implicit equation 3.7 when $\mu_H > \mu_L$. If the marginal cost of high-quality firm is equal to the marginal cost of low-quality firm, then the same optimal advertising level $A^*$ is derived. Hence, pooling equilibrium exists. Moreover, similar with the Normal distribution part, there also exists a possibility of distorted price for high quality firm. If high quality firm can distort their price, then it is possible to get a different result.

ii. If $\mu_H < \mu_L$ , then $\alpha_H < \alpha_L$. In this case, the factor $\frac{(1-A)^{\alpha_H-\alpha_L-1}}{A^{\alpha_H-\alpha_L+1}}$ will be negative, and so will the $V_A$. This means the profit function decreases in advertising level. Thus, A = 0 is the optimal level of advertising for both types. Hence, this is a pooling equilibrium.

### *3.2.2 Asymmetric marginal costs*

Similar to the normal distribution part, this subsection will analyze what would happened if the marginal costs of different types are different.

i. If $c(H) > c(L)$, then $P - c(H) < P - c(L)$. Thus, $(1+\ell)(P - c(H))\mathrm{m}P\,\frac{(1-\gamma)}{\gamma}\,(\alpha_H - \alpha_L) < (1+\ell)(P - c(L))\mathrm{m}P\,\frac{(1-\gamma)}{\gamma}\,(\alpha_H - \alpha_L)$. As a result, $\frac{(1-A_H^*)^{\alpha_H-\alpha_L-1}}{{A_H^*}^{\alpha_H-\alpha_L+1}} > \frac{(1-A_L^*)^{\alpha_H-\alpha_L-1}}{{A_L^*}^{\alpha_H-\alpha_L+1}}$ and then $A_H^* < A_L^*$. The high-quality firm's optimal advertising level is smaller than the low-quality firm's advertising level. This result is consistent with Schmalensee (1978).

ii. If $c(H) < c(L)$, then $P - c(H) > P - c(L)$. Hence, $(1+\ell)(P - c(H))\,\mathrm{m}P\,\frac{(1-\gamma)}{\gamma}\,(\alpha_H - \alpha_L) > (1+\ell)(P - c(L))\mathrm{m}P\,\frac{(1-\gamma)}{\gamma}\,(\alpha_H - \alpha_L)$. As a result, $\frac{(1-A_H^*)^{\alpha_H-\alpha_L-1}}{{A_H^*}^{\alpha_H-\alpha_L+1}} < \frac{(1-A_L^*)^{\alpha_H-\alpha_L-1}}{{A_L^*}^{\alpha_H-\alpha_L+1}}$ and then $A_H^* > A_L^*$. In this case, the low-quality firm's optimal advertising level is smaller than the high-quality firm's advertising level. This result is consistent with Nelson (1974).

## 4. <u>Result and Conclusion</u>

Two tables are given below to show the conclusion of this report: the first one is for a discrete consumer belief b, and the second one is for a continuous consumer belief b.

**Table 1** Discrete consumer belief b

| | **Bagwell's model** | **My model** |
|---|---|---|
| **Demand** | D(P,A, b) > 0 | D (P, b) = a – m $\frac{P}{b(A)}$ |
| **Advertising Type** | Dissipative advertising<br>$D_A = 0$ | non-dissipative advertising<br>if $b_A \neq 0$ then $D_A \neq 0$ |
| **Marginal cost and Result** | $C_H > C_L$<br>High-quality firm can signal its quality in least cost separating equilibrium | $C_H > C_L$<br>High-quality firm can signal its quality in least cost separating equilibrium conditionally |
| | $C_H < C_L$<br>least cost separating equilibrium does not exist | $C_H < C_L$<br>least cost separating equilibrium does not exist |

From the table 1, it is clear to see that if the advertising level changed from dissipative advertising to non-dissipative advertising, the result remains similar. Both models require additional conditions for the existence of least cost separating equilibrium.

**Table 2** Continuous consumer belief b

| | **Normal Distribution** | **Beta Distribution** |
|---|---|---|
| **Price** | $P_H = P_L$ | $P_H = P_L$ |
| **Demand** | D (P, b) = a – m $\frac{P}{b(A)}$ | D (P, b) = a – m $\frac{P}{b(A)}$ |
| **Advertising type** | If $\mu_H > \mu_L$, demand enhancing advertising | If $\mu_H > \mu_L$, demand enhancing advertising |
| **Marginal cost, advertising level and Result** | $c(H) = c(L), \mu_H > \mu_L$<br>the pooling equilibrium exists with identical advertising level for both types | $c(H) = c(L), \mu_H > \mu_L$<br>the pooling equilibrium exists with identical optimal advertising level for both types |
| | $c(H) = c(L), \mu_H < \mu_L$<br>the pooling equilibrium exists with a corner solution A=0 for both types | $c(H) = c(L), \mu_H < \mu_L$<br>the pooling equilibrium exists with a corner solution A=0 for both types |
| | $c(H) > c(L)$<br>$A_H^* < A_L^*$<br>Low-quality firms advertise more, parallel with Schmalensee's result. | $c(H) > c(L)$<br>$A_H^* < A_L^*$<br>Low-quality firms advertise more, parallel with Schmalensee's result. |
| | $c(H) < c(L)$<br>$A_H^* > A_L^*$<br>High-quality firms advertise more, parallel with Nelson's result | $c(H) < c(L)$<br>$A_H^* > A_L^*$<br>High-quality firms advertise more, parallel with Nelson's result |

From the table 2, it demonstrates that the pooling equilibrium exists in a two-period dynamic model where the consumer belief b(A) is a continuous variable and the advertising level A is a random variable drawn from Normal distribution and Beta distribution respectively.

Both Normal and Beta distributions lead to the same outcome. When the marginal costs for both high-quality firms and low-quality firms are equal, the pooling equilibrium exists and an optimal advertising level A* that maximize the profit can also be found if the average advertising levels of high-quality firms are higher. As opposed to this, when the marginal costs for both high-quality and low-quality firms are equal and the average advertising level of low-quality firms are greater, the pooling equilibrium still exists but only with a corner solution A = 0 to maximize the profit.

If the marginal cost of high-quality firm is higher, the optimal advertising level of high-quality firm is lower than the low-quality firm's advertising level, which means the low-quality firms advertises. This is consistent with Schmalensee (1978). Conversely, if the marginal cost of low-quality firm is bigger than the marginal cost of high-quality firm, then the optimal advertising level of low-quality firm is lower than the optimal advertising level of high-quality firm, so the high-quality firm advertises more. This is consistent with Nelson's result.

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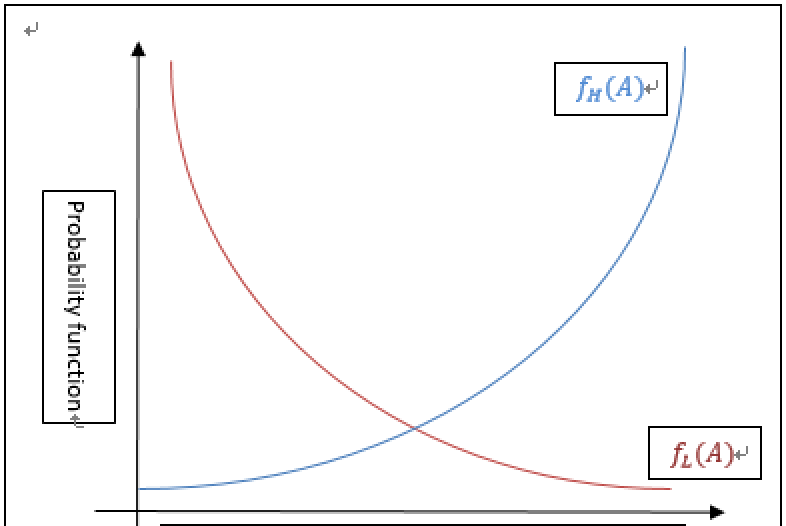
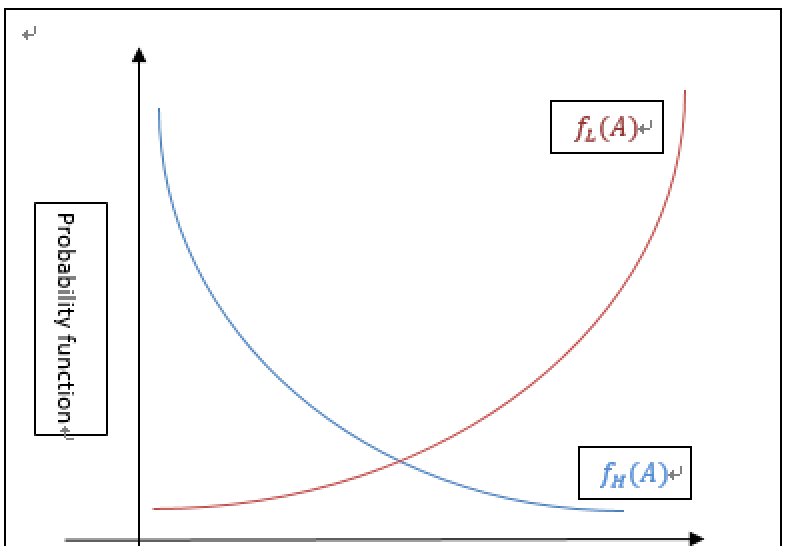
# Appendix




**Graph 1**




**Graph 2**

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