A Convexity Adjusted Duration Gap Model to Measure Interest Rate Risk

Application to a Hypothetical Small Bank

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ABSTRACT

The price/yield relationship of a debt instrument, without any embedded option, is convex and it is theoretically well established that duration of the debt instrument provides only the first order (linear) effect, on the price in response to unexpected changes in the yield. Therefore, the duration model overestimates the decline in the price when there is a large increase in the yield and underestimates the increase in the price when there is a large decrease in the yield. The convexity measure of the debt instrument reduces the error of overestimation and underestimation by providing the second order (curvilinear) effect on the price of large changes in the yield. Despite this recognition of the role of the convexity measure, the convexity adjustment to the duration gap model has been neglected in the extend literature to quantify interest rate risk of banks (see, for example, Beets (2004), Entrol et al. (2009), the Basel Committee on Banking Supervision. (2004), among others). This study proposes a convexity adjusted duration gap model to quantify interest rate risk of a bank. In addition, recognizing that the yield curve is normally upward sloping, not flat as is normally assumed, the study uses different interest rates and yields for different assets and different liabilities of the bank. Finally, it permits different adoption of changes in interest rates to different classes of assets and liabilities. The study also presents an application of the model so developed to a hypothetical bank to quantify its interest rate risk and strategies to reduce interest rate risk in the context of a convexity adjusted duration model.

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1.0 Introduction

The interest rate risk of a bank arises from several factors including: (i) The maturities mismatch between assets and liabilities of a bank as it allocates funds to credit instruments whose contractual maturities are long-term while it raises funds through deposits whose contractual maturities are short-term. (ii) The basis value risk due to imperfect correlation in the adoption of changes in interest rates to assets and liabilities. For example, if interest rates rise, a bank may be quick to raise interest rate on its loan, and may delay raising interest rate on its discretionary rate instruments, and non-maturing deposits, etc. (iii) The yield curve risk that arises from the effects of changes in the shape and slope of the yield curve on the economic value of the bank. (vi) The embedded options in assets and liabilities arising from prepayment provision, early redemption of term deposits without penalty, current accounts and so on, (v) The off-balance sheet activities such as credit substitutes and positions in financial derivatives.

Hirtle(1996) argues that interest rate risk is one of the most important risks faced by banks in their role as financed intermediaries. The Basle Committee on Banking Supervision in its consultative proposal, entitled, “Measure of Banks’ Exposure to Interest Rate Risk” (1993) recognized that after credit risk which was the basis of designing the initial risk-weighted capital adequacy requirements, the interest rate risk is a significant risk to banks, requiring a common approach to measuring it so that supervisory actions can be recommended for banks that show higher levels of interest rate risk. The Basle Committee on Banking Supervision in 2004 proposed a standardized framework to value interest rate risk. Then, the historical episodes of banking crisis such as the Savings and Loan Associations Crisis in late 1980s in which 13.5% of 4,000

Savings and Loans banks failed indicate the importance of interest rate risk. The reason of mass failures of Savings and Loans banks was that their deposits had be to re-priced frequently in rising interest rate environments while on their main assets: fixed-rate mortgage loans where interest rates could not be raised.

Due to the importance of interest rate risk, banks calculate what is known as the duration gap of their interest sensitive assets and interest sensitive liabilities by estimating the difference between the market values weighted average of durations of assets and the market values weighted average of durations of liabilities, adjusted for the leverage ratio of the bank. For most banks, the duration gap is negative, therefore, any upward interest rate shock results in a decline in the value of the bank.

The main advantage of the duration gap model is that it provides a single number to measure the interest rate risk of a bank, but as is generally recognized that the application of this model in real world situation faces a series of complex issues. (See Basle Committee on Banking Supervision (April, 1993)). Firstly, except trading assets which are marked to market, non-traded assets and liabilities are in the book value terms, while duration gap model requires the market valuations. Secondly, there are items in a bank’s balance sheet whose maturities differ from their contractual terms making their actual maturities uncertain and these items include mortgage loans with its prepayment-provision, term deposits that provide an option to the depositor to withdraw the deposit before the final maturity date, etc. Thirdly, there are saving deposits and demand deposits on which interest rates paid are less responsive to changes in the market yield permitting the bank option to select timing and to the adoption of interest rate changes. Fourthly, within each class of assets or liabilities, there are a host of differences in terms of interest rates paid, floating rate versus fixed rate instruments, different interest rate reset periods for floating rate instruments, different payout periods and different relevant market yields. Finally, there are off-sheet assets and liabilities of a bank which need to be converted to equivalent on balance sheet assets and liabilities.

The point is that the calculation of the duration gap for an actual bank is a difficult task, requiring individual detailed data on bank's assets and liabilities, which are not publicly available, obtaining estimates of behavioral maturities instead of contractual maturities for many instruments and obtaining estimates of the cash flows time paths of instruments over their relevant maturities, etc.

The objective of this study to show that adding the convexity measure to the duration gap of a bank derived under extremely simplified assumptions reduces the error of prediction of the effect on the equity of the bank of on adverse interest rate shock. Secondly, unlike the same interest rate used for different assets and liabilities, the study will use different interest rates for different classes of assets and liabilities of a bank. Thirdly, it will permit differential adjustments of interest rates paid on the individual classes of assets and liabilities in response to an interest rate shock. Finally, in the context of a convexity-adjusted duration model the study will provide strategies in interest rate risk. It will also provide an example of hedge against interest rate risk by using the interest rate future contracts.

This study contributes to the extant literature by providing an emphasis of the incorporation of the convexity measure in the duration gap model to reduce the error in predicting the effect on the equity value of a bank to an adverse interest rate shock. In the context of the mandatory capital adequacy requirements of a bank, even a small correction of error can have a meaningful effect. Additionally, the use of different yields and differential adjustments is expected to add value to the textbook treatment of the duration gap model as a measure of interest rate risk of a bank.

The rest of the study is structured as follows: The next section will provide a literature review on the duration gap model. In section 3, the convexity adjusted duration gap model of the study will be presented. Section 4 provides the numerical illustration of the model and it will also present proposals of managing interest rate risk. Section 5 will give summary and conclusion of the study.

2.0 Literature Review

Under the auspices of the Bank of International Settlements, the Basel Committee on Banking Supervision met in 1988 and based on credit risk weighted assets, it designed capital adequacy requirements of banks. In Basel Accord 1988, interest rate risk was not taken into account. However, the Basel Committee in its consultative proposal entitled" Measurement of Banks' Exposure to Interest Rate Risk (April, 1993) stated that "All the members of the Basel Committee regard interest rate risk as a significant risk which banks and their supervisors need to monitor carefully." In the proposal, the Committee recommended duration gap as a measurement model of interest rate risk of banks. In this proposal, the Committee suggested to place all interest sensitive assets, liabilities and off-balance sheet positions into one of thirteen time-bands based on their maturities or re-pricing characteristics. The position in each time-band be netted and the resulting net position for each time-band be item weighted by an estimate of its duration. It also suggested adjustment of duration weights to reflect relative volatility of interest rate across the term structure.

For the measurement of interest rate risk, regulators and banks employ a variety of other techniques, besides the duration gap technique (Feldman and Schmidt, 2000). One of these techniques is known as the gap analysis in which assets and liabilities that are interest sensitive within a given time-band are netted out, and the effect on spread income within the band in response to an interest rate shock is treated as a measure of interest rate risk (Schiffer, 1991). This approach, however, does not provide a single statistic that indicates the interest rate risk. Also, other problems with this approach are that it bases its analysis on the book values, not on market values; secondly, it ignores different re-pricing within the time-band of rate sensitive assets and liabilities. (that is the problem of over-aggregation); and, finally, it also ignores unequal interest rates on assets and liabilities and possible change in net worth of the bank.

Another technique to measure interest rate risk of banks is known as the simulation-scenario analysis in which changes in the banks' earnings and economic value under various alternative interest rate scenarios are modelled (Payne et al, 1999; Schaffer, 1991). The drawback of this analysis is that it requires the detailed payments data of all the credit instruments which is difficult to obtain.

There are other comparable models, such as the Economic Value Model, used by the Federal Reserve to quantify the interest rate risk of U.S. commercial banks (see, for example, Houptant Embersil, 1991; Sierrea and Yeager, 2004) and models that analyze the interest rate sensitivity of stock return of the banks that are publicly traded (see, for example, Stone, 1974; Cgaja et al, 2009a, 2009b).

In this report, however, we deal only with the duration gap in a simple banking situation in order to show the effect of the convexity measure to quantify the interest rate risk. Entrpo et al (2009) note that in one of the reports of the Basel Committee in 2004, entitled “Principles of Management and Supervision of Interest Rate Risk,” the Committee provides a standardized framework to measure the interest rate risk by banks. The framework, so provided, is based on specific assumptions related to number and boundaries of time-bands of assets, liabilities and off-balance sheet positions and distributions of maturities within each band. Entrop et al (2009) find that specific assumptions of the standardized framework of estimating the interest rate risk are restrictive. As a consequence, for some banks, the assumptions cannot apply and thereby the supervisors are likely to make a mistake on quantifying the interest rate risk and react inappropriately. They, instead, allow different boundaries of the time-bands, different distribution within the time-bands, different amortization rates and different coupon rates. According to them, this relaxation of the Committee’s assumptions generalizes the standardized framework which can be applied to any type of the bank at any time and in any country.

Beets (2004) presents the standard duration gap model for a bank which has a single asset and a single liability and the same interest rate for both asset and liability. In this simple framework, it suggests the use of interest rate derivatives to hedge the interest rate risk, whose level is determined by the duration gap model. The use of interest rate derivative in order to hedge interest rate risk has been also suggested by Brewer and Moser (2001).

Liu et al (2004) applied the valuation model of Rabinovitch (1989) that models a bank’s equity as a call option. They show how a bank’s total risk can be decomposed into interest rate risk and non-interest rate risk. Instead of using a constant risk-free interest rate, following Merton (1973) who introduced a stochastic interest rate process, they introduce a stochastic process described in Vasicek (1977) and the resulting equation permitted them to decompose total risk into interest rate risk and non-interest rate risk. Then, they introduce the Z-score as a measure of distance-to-default, and applied the Z-Score Model to six publicly traded Canadian banks during 1982-2002.

Dzmuranova and Teply (2015) presents a theoretical framework to estimate duration of non-maturing instruments. Demand deposits are non-maturing instruments with no specified liquidity and interest rate behavior by a contract. They identify typical non-maturity instruments on the asset side as overdrafts and credit cards while on the liability side as current deposits and savings deposits. Their paper provides a procedure to estimate maturity and duration of demand deposits.

Demand deposits are withdrawn on demand, which implies that legal duration of demand deposits is one day but in fact that being as transaction deposits and netting of withdrawals from the additions to demand deposits keeps overtime a stable core balance of these deposits, the actual duration of demand deposits is significantly longer than one day. Dzmuranova and Teply (2015) use historical volumes of demand deposits and benchmark market rates, to create replication portfolios which, using the stochastic process proposed in Vasicek (1997) to determine the benchmark market rates, provide an estimate of duration of demand deposits. They also noted that there are numerous interest rate models used by other authors to model duration of demand deposits (for example, see Kalkbreuer and Willing, 2004; Nystrom, 2008; Hull and White, 2008)

Despite the fact that most of the papers reviewed above have covered the duration gap approach as a measure of interest rate risk of banks, none of them has suggested that the convexity adjustment can reduce the error of prediction about the change in the economic value of a bank. Licak (2004) is the only paper which recommends the incorporation of the convexity adjustment as it states on its p. 6 that “for the price of an instrument it holds true that in the case of smaller interest-rate changes the changes of prices are the same as in the case of a fall or rise in interest rates. On the other hand, in the case of larger shifts in interest rates, the changes in the prices of an instrument in the case of a rise or fall are different. Duration does not fulfill this second condition, for which it holds true that in the case of however large changes in interest rates the subsequent changes in price are equal. For this reason duration as such is an appropriate indicator of interest-rate risk only in the case of small interest rate changes. This deviation — the degree of convexity, which arises in the case of larger interest-rate changes, may be found through a second derivative. The degree of convexity together with duration gives us the change in the value of the financial instrument also in the case of larger movements in interest rates.”

However, despite identifying a need to consider convexity together with the duration gap, Licak(2006) does not suggest a convexity adjusted duration gap model to quantify interest rate risk of banks. This project report introduces such a model in its next section. It will also consider classifications of assets into short-term assets and long-term assets and of liabilities into short-term liabilities and long-term liabilities. Additionally, the proposed model will apply different yields to different classes of assets and liabilities and permit different adjustment of nominal rates or coupon rates on credit instruments in the face of an interest rate shock. Finally, the study shows the strategies to manage interest rate risk under the framework of a convexity adjusted duration gap model and shows the use of the interest rate futures to hedge interest rate risk.

3.0 The Model

3.1 Introduction and Assumptions

The slope of the price/yield curve of a debt security at a given level of yield is represented by its duration while the change in the slope of the price/yield curve is given by convexity. In other words, the first order effect of change in the price of a debt security, d_p, to a very small change in its yield, d_k, which is d_p/d_k, is measured by duration while the convexity effect represents the second order effect, that is d_p^2/d_k^2. Therefore, incorporation of the convexity adjustment to duration gap of a bank should show an improvement in predicting interest rate induced price changes of assets liabilities and equity. The convexity adjustment is crucial for a bank to reduce errors for two main reasons: (i) changes in interest rates or yields are not infinitesimal in practice, and (ii) the dollar magnitudes of portfolios of both assets and liabilities are large of a typical bank.

To introduce a convexity adjusted duration gap model to quantify interest rate of a bank, we proceed sequentially from the consideration of a hypothetical simple bank to the consideration of a somewhat more complex bank. For an actual bank, the estimation of a convexity adjusted duration gap of the bank is extremely difficult for the reasons which are given in the following.

First of all, the overall dollar size of each class of assets or each class of liabilities represents the aggregation where within the class, there are different contractual maturities, different interest rates, different prevailing market rates and some assets have embedded options while others do not. This means that within each asset class, durations of individual assets in the class are required. For example, for a typical

Canadian bank, the classes of assets are usually as follows: deposits with other institutions, trading assets, securities under repos and securities borrowed, restructured securities, residential mortgage, loans: personal loans, credit cards, business and government loans, etc. And the same complexity applies to the liabilities side. The classes of liabilities include: personal demand deposits, notice deposits, non-term savings deposits and term deposits. A similar deposit classification exists for business and government sectors. There are also deposits made by other financial institutions. There are also other interest bearing liabilities.

Secondly, due to embedded options in some assets and liabilities, the actual maturities of those assets and liabilities differ from the contractual maturities. For example, a prepayment option in a residential mortgage loan requires a forecast of interest rates path to estimate the behavior of the mortgagors to prepay outstanding loans in each interest rates environment.

Thirdly, except for trading assets where mark-to-market valuation methodology is used, other instruments in the balance sheet use book values while the calculation of duration and convexity require the market valuation.

Fourthly, the balance sheet of a bank is a consolidated balance sheet with some assets and liabilities denominated in foreign currencies where countries of different currencies may exhibit the different interest rates. Also, some positions in foreign currency assets may be offsetting each other while others do not. From that standpoint of view of calculations of duration and convexity adequate data on individual assets and liabilities in different currencies are required, which seems to be a difficult task.

Fifthly, for non-mature assets and liabilities, for example, overdrafts and credit cards on the asset side, current notice and non-term savings deposits on the liability side, estimates of their behavioral maturities are required and these estimates base on the appropriate stochastic processes to determine the market rates paths and replicating portfolios.

Sixthly, for the calculation of durations and convexity of off-balance sheet assets and liabilities, credit substitutes and derivative instruments need to be converted into notional on-balance sheet positions. This may be possible due to the availability of the regulatory conversion factors, however, one also needs to learn the which of derivatives used for hedging purpose and which are not.

The purpose of this study is to build a simplified bank model and extend the duration gap model by using convexity to reduce the errors of prediction of changes in the economic value of a bank. Also, in addition to the textbook treatment of duration gap analysis in which the same interest rate is used for both liabilities and assets, this study uses different interest rates for different assets and liabilities of the bank. The study will also categorize the assets and liabilities into different classes based on the terms. Finally, the study will cover hedging strategies, including the use of the future contracts. There will be data derived from a real bank financial performance report as a reference to interest rates.

For the convenience of calculation purposes, we will make the following assumptions.

i. For each of all the classes of assets and each of all the classes of liabilities, the specific interest rate earned or paid is known.
ii. For every class, assets or liabilities, the actual average maturity is known.
iii. The timing patterns of all assets and liabilities classes are known.
iv. The book values of all classes of assets and liabilities are given.

v. The off-balance sheet positions of the bank do not exist.

3.2 A case of a bank with only one Asset and One Liability

We will initiate our model with a very simple situation where a bank has only one asset, denoted by A, and only one liability, denoted by L. Additionally, it is assumed that the maturity of A is N_A years, and the maturity of L is N_L years, (with $N_A > N_L$. The interest rate earned on A is i_A per annual and the interest rate paid on L is i_L annually. The prevailing market yields are k_A per year on A and k_L per year on L. Finally, for the sake of simplicity of calculations of durations and convexities of A and L, it is assumed that on A, the payment will be paid by the borrowers a single amount after N_A years and the bank will pay on L a single amount to the depositors after N_L years.

Suppose the bank has the following book value balance sheet.

Table 1

Asset ($)	Liability and Equity ($)
A'	L'
	E'
A'	$L' + E'$

Table 1: The Book Value Balance Sheet of a Hypothetical Bank

Since the duration and convexity measures require the market valuation balance sheet. We will obtain the market value balance sheet as following:

$$A = A'(1+ i_A)^{NA}/(1+ k_A)^{NA}$$

$$L = L'(1+ i_L)^{NL}/(1+ k_L)^{NL}$$

$$E = A - L \quad (1)$$

Table 2

Asset ($)	Liability and Equity ($)
A	L
	E
A	L+ E

Table 2: The Market Value Balance Sheet

Now we are going to use "d" as the change, the change in the equity or economic value of the banks is given by:

$$dE = dA - dL \quad (2)$$

Noting that the duration D of a single payment credit instrument is equal to its maturity, the duration of A is equal to maturity of A, $D_A = N_A$, maturities of L, D_L is N_L. From the duration model, it is common knowledge that any credit instruments with its current market value of P, and its current relevant yield, k, the change in the price of the instrument, d_p, in response to a change in its yield, d_k is:

$$d_p = -D\frac{dk}{1+k}P \quad (3)$$

Now using the relationship in equation (2), we can get the duration gap model as:

$$d_E = -D_A\frac{d_{KA}}{(1+k_A)} * A - (-D_L\frac{d_{KL}}{(1+K_L)} * L)$$

$$d_E = -(D_A\frac{d_{KA}}{(1+k_A)} - D_L\frac{d_{KL}}{(1+K_L)} * \frac{L}{A}) * A \quad (4)$$

This is the standard duration gap model except that $k_A \neq k_L$ and d_{KA} can be different from d_{KL}. The duration gap refers the difference between the two items in the brackets in equation (4).

The convexity adjustment of the duration gap in equation (4) requires the convexity measure for both A and L. We denote the convexity measure of A by CX_A and convexity of L is CX_L. As for the calculation of a convexity measure of a credit instrument whose current value is P and the current yield is k, an infinitesimal rise and fall of k is required, which analysts assume to be 1.0 basis point change in k. By definition, the convexity parameter of the credit instrument is given by[1,2],

$$CX = (P_+ + P_- - 2P) / P * (0.0001)^2 \tag{5}$$

Where P_+ is the market value of the credit instrument due to a 1 basis point decrease in its k and P_- is the market value due to a 1 basis point increase in k.

Using equation (5), CX_A and CX_L are given as:

$$CX_A = (A_+ + A_- - 2A)/A * (0.0001)^2 \tag{5}$$

′

$$CX_L = (L_+ + L_- - 2L)/L * (0.0001)^2 \tag{5″}$$

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[1] See Chapter 7 of Falooggi: Fixed Income Analysis, AIMR, 2000

[2] It may be noted that equation (5) can also be written as:

$$CX = 10^8 \left[\frac{\Delta P_+}{P} + \frac{\Delta P_-}{P}\right]$$

Recognizing that convexity measure represents the second order effect (Taylor Series), we get the convexity adjusted duration gap model in the simplified situation of the bank as follows:

$$dE = -\left(D_A \frac{d_{KA}}{(1+k_A)} - D_L \frac{d_{KL}}{(1+K_L)} \frac{L}{A}\right) * A$$

$$+ \frac{1}{2}\left(CX_A * d_{KA}{}^2 - CX_L * d_{KL}{}^2 \frac{\mathbf{L}}{\mathbf{A}}\right) * A \qquad (6)$$

Equation (6) provides the quantification of interest rate of a bank. For a bank, the duration gap is usually positive mainly for the reason that it lends long-term and borrows short-term. This means that an anticipated upward drift of yields will adversely affect the bank's equity position. Next, we show the size of reduction in error in predicting the change in equity by a simple illustration. Equation (6) provides the quantification of interest rate of a bank.

Now let's have a look at an example on convexity adjusted duration gap model. It is assumed that the book value of A and L of a bank are respectively as \$96.6797 million and \$ 91.54902 million which give the book value of E as \$ 5.13058 million. The maturities of assets and liabilities are assumed to be 5 years and 1 years respectively, that is, N_A=5 and N_L=1. Interest rates and current yields are assumed to be: i_A=0.04; i_L=0.02; k_A=0.033 and k_L=0.15. Given that single payment assumption on the assets and liabilities, the market values of assets and liabilities are:

$$A = 96.6796 * (1.04)^5/1.033^5 = \$ \ 100 \text{ m}$$

$$L = 91.54902 * 1.02/1.015 = \$ \ 92 \text{ m}$$

Therefore, the market value balance sheet of the bank is:

Asset ($;million)	Liability and Equity ($:million)
A=100	L=92 E=8
A=100	L+ E=100

Given that $D_A = N_A = 5$ and $D_L = N_L = 1$, CX_A and CX_L are calculated as follows:

After 5 years, the single payment of A = $96.6796 * (1.04)^5 = 117.625516$. A_+ and A_- are respectively calculated assuming that $k_A = 0.033$ declines and increases by 1 basis point.

$$A_+ = 117.625516/1.0329^5 = \$\ 100.0484137 \text{ m}$$

$$A_- = 117.625516/1.0331^5 = \$\ 99.9516083 \text{ m}$$

Similarly, L_+ and L_- are obtained letting $k_L = 0.015$ decrease and increase by 1 basis point. The single payment on L after I year is: L =91.54902 ∗ 1.02 = 93.38

$$L_+ = 93.38/1.0149 = \$\ 92.0090649 \text{ m}$$

$$L_- = 93.38/1.0151 = \$\ 91.9909369 \text{ m}$$

Now let's use equation (5) ′ , the convexity measuring of assets CX_A is calculated as:

$$CX_A = (100.0484137 + 99.9516083 - 200)/100 * (0.0001)^2 = 22$$

Similarly, using equation (5)″, the convexity measure of liabilities, CX_L is:

$$CX_L = (92.0090649 + 91.9909369 - 2 * 92)/92 * (0.0001)^2 = 1.9565$$

Then, ΔA due to an increase of 100 basis point and ΔL due to an increase of 80 basis points reflecting the expectation of a more steeper upward sloping yield curve:

$$\Delta A = -\left(D_A \frac{d_{KA}}{(1+k_A)} - \frac{1}{2} CX_A * d_{KA}{}^2\right) A$$

$$= -\left(5 * 0.01/1.033 - 0.5 * 22 * 0.01^2\right) 100$$

$$= -4.4804 + 0.11$$

$$= -\$\ 4.7303$$

$$\Delta L = -\left(D_L \frac{dkL}{(1+kL)} - \frac{1}{2} CX_L * d_{KL}{}^2\right) L$$

$$= -\left(1 * 0.008/1.015 - 0.5 * 1.9565 * 0.008^2\right) 92$$

$$= -0.725129 + 0.00576$$

$$= -\$\ 0.71936$$

Therefore,

$$\Delta E = \Delta A - \Delta L$$

$$= -4.7303 - (-0.71936) = -4.0109$$

The actual declines in A and L, given their future payments of \$117.625534 m after five years and \$ 93.38 m after one year respectively, are:

$$A' = 117.625534/1.043^5 = \$\ 95.2971836 \text{ m}$$

$$L' = 93.38/1.023 = \$\ 91.280547 \text{ m}$$

This gives the ΔA and ΔL,

$$\Delta A = 95.2971836 - 100 = -\$\ 4.702816 \text{ m}$$

$$\Delta L = 91.280547 - 100 = -\$\ 0.719453 \text{ m}$$

Thus,

$$\Delta E = \Delta A - \Delta L$$

$= - 4.702816 - (- 0.17945)$

$= - \$ 3.983361$ m

The above results are given in Table 3 below, given book value of A=$96.6797 million, L=$91.54902, i_A=0.04; i_L=0.02; k_A=0.033 and k_L=0.15 per annual with single payment. (Figures are in $ million):

Table 3

	Actual Declines	The Duration Gap Model: Declines	Convexity Adjusted Duration Gap Model: Declines
ΔA due to 1% increase in k_A	–$4.702816	–$4.8403	– $ 4.7303
ΔL due to 0.8% increase in k_L	–$0.719453	–$0.725129	–$ 0.71936
ΔE	–$3.983361	–$4.7303	–$4.0109

Table 3: The Reduction in Error due to Convexity Adjustment

As expected, the convexity adjusted model reduces the error of over-prediction of the declines in assets, liabilities and equity. Overall, the duration gap model over-predicts the decline by $ 0.74694 million while the convexity adjusted duration model over-predicts the decline only by $0.027589 million. The error is corrected by the convexity is $0.71935 m and this can be significant for a bank which is satisfying the capital adequacy requirements only at margin.

3.3 The Convexity Adjusted Duration Gap model for a Hypothetical Bank with two classes of Assets and two classes of Liability

3.3.1 Notation and Assumptions

In this subsection, we consider two classes of assets and two classes of liabilities of a hypothetical bank with reference to a real bank to derive the convexity adjusted duration gap model. To keep focusing on the development of the model, the use of different interest rates and different market yields for classes of assets and liabilities on the uneven shift in the yield curve, a series of assumptions will be made below:

The notation used in this subsection are:

A_s=short-term Assets

A_T=long-term Assets

L_s=short-term Liabilities

L_T=Long-term Liabilities

i_{AS}=the annual interest rate the bank receives on its short-term assets

i_{AL}=the annual interest rate the bank receives on its long-term assets

i_{LS}=the annual interest rate the bank pays on its short-term liabilities

i_{LL}=the annual interest rate the bank pays on its long-term liabilities

k_{AS}=the current annual market yield on bank's short-term assets

k_{AL}=the current annual market yield on bank's long-term assets

k_{LS}=the current annual market yield on bank's short-term liabilities

k_{LL}=the current annual market yield on bank's long-term liabilities

N_{AS}=the maturity of short-term assets (in years)

N_{AL}=the maturity of long-term assets (in years)

N_{LS}=the maturity of short-term liabilities (in years)

N_{LL}=the maturity of long-term liabilities (in years)

Assumptions:

With respect to the cash flow paths, we simply assume:

i. Each class of assets or liabilities has a specified fixed maturity.

ii. There are no embedded options.

iii. On all the instruments in each class, only one interest rate is received (for an asset) or paid (for a liability).

iv. Over the definite maturity, the cash flow path of each class is given by an annuity.

v. All the relevant interest rates are known and they stay the same over the definite maturities.

vi. To calculate the market values, it will be assumed that we know the current yield.

3.3.2 The Convexity Adjusted Duration Gap Model for Two Classes of Assets and Two Classes of Liabilities

Following the framework of section 3.2 above, the convexity adjusted duration gap model for two assets and two liabilities is given by:

$$\Delta E = [-(D_{AS}\frac{d_{KAS}}{(1+k_{AS})} - \frac{1}{2}CX_{AS} * d_{KAS}^{2})A_S$$

$$-(D_{AL}\frac{d_{KAL}}{(1+k_{AL})} - \frac{1}{2}CX_{AL} * d_{KAL}^{2})\, A_L] -$$

$$[-(D_{LS}\frac{d_{KLS}}{(1+k_{LS})} - \frac{1}{2}CX_{LS} * d_{KLS}^{2})\, Ls$$

$$-(D_{LL}\frac{d_{KLL}}{(1+k_{LL})} - \frac{1}{2}CX_{LL} * d_{KLL}^{2})\, L_L\,] \tag{7}$$

According to the annual report of Bank of Nova Scotia (Appendix), the interest rates for different classes of assets and liabilities in respectively 2018 and 2017 are shown on Table 4 below:

Table 4

Asset	($ million)	Weighted interest rate 2017	Weighted interest rate 2018
ST Asset	148.6	0.53%	0.88%
LT Asset	694.7	3.11%	3.45%
Other Assets	102.4		
Total Asset	945.7	2.34%	2.67%
Liability			
ST Liability	0	0.00%	0.00%
LT Liability	812	1.09%	1.44%
Other Liabilities	69.60		
Equity	64.1		
Total Liability	881.6	0.93%	1.23%

Table 4: The interest rate of long-term and short-term Liabilities and Assets of Bank of Nova Scotia in 2018 and 2017

Based on the data shown above, we are going to make assumptions that can simplify our calculations. The assumptions are as follows:

Book Value Balance Sheet:

Asset ($ million)	Liability and Equity ($ million)
$A_s'=20$	$L_s'=12$
$A_L'=80$	$L_L'=78$
	$E'=10$
$A'=100$	$L'+E'=100$

Interest rates and yields:

$i_{AS}=0.015$	$i_{AL}=0.045$
$i_{LS}=0.005$	$i_{LL}=0.020$
$k_{AS}=0.02$	$k_{AL}=0.04$
$k_{LS}=0.0075$	$k_{LL}=0.015$

Maturities:

$N_{AS}=0.5$ years

$N_{AL}=5$ years

$N_{LS}=0.25$ years

$N_{LL}=2$ years

Cash Flows:

A_s: Monthly equal amounts

A_L: Six-monthly equal amounts

L_s: Bi-weekly equal amounts

L_L: Quarterly equal amounts

Now let's move forward to the calculation process. Firstly, we are going to calculate the relevant rates and yields:

A_s: monthly $i_{AS} = 0.015/12 = 0.00125$, $k_{AS} = (1.02)^{1/12} - 1 = 0.001652$

A_L: six monthly $i_{AL} = 0.045/2 = 0.0225$, $k_{AL} = (1.04)^{1/2} - 1 = 0.01980$

L_s: bi-weekly $i_{LS} = 0.005/26 = 0.000192$, $k_{LS} = (1.0075)^{1/26} - 1 = 0.002874$

L_L: Quarterly $i_{LL} = 0.02/4 = 0.005$, $k_{LL} = (1.015)^{1/4} - 1 = 0.003729$

Then, we will calculate the annuities:

$$A_s: A = 20/[(1 - 1.00125^{-6})/0.00125]$$
$$= 20/5.973837$$
$$= 3.3479318$$

$$A_L: A = 80/[(1 - 1.0225^{-10})/0.0225]$$
$$= 80/0.1127877$$
$$= 9.0230146$$

$$L_s: A = 12/[(1 - 1.000192^{-6.5})/0.000192]$$
$$= 12/6.4953225$$
$$= 1.8474833$$

$$L_L: A = 78/[(1 - 1.005^{-8})/0.005]$$
$$= 78/7.822959$$
$$= 9.970652$$

Next, according to the annuities, maturities and market yields, calculate the current market values:

$A_s = 3.3479318 * [(1-1.001652^{-6})/0.001652]$

$=3.3479318 * 5.96546$

$=19.97195412$

$A_L = 9.0230146 * [(1-1.019804^{-10})/0.019804]$

$= 9.0230146 * 8.9918035$

$= 81.13317462$

$L_S = 1.8474833 * [(1-1.0002874^{-6.5})/0.0002874]$

$=1.8474833 * 6.493$

$=11.99570967$

$L_L= 9.970652*[(1-1.003729^{-8})/0.003729]$

$=9.970652* * 7.8674077$

$=78.44318425$

Therefore, the market value balance sheet is:

Asset ($;million)	Liability and Equity ($:million)
A_s=19.97195	L_s=11.9957
A_L=81.1332	L_L=78.4432
	E=10.6663
A=101.1052	L+ E=101.1052

Now, we are calculating the durations and convexities:

According to the short-cut formula for duration, which is:

$$D = \frac{i(1+k)\left[\frac{1-(1+k)^{-n}}{k}\right]+n(k-i)/(1+k)^{n}}{i+[(k-i)/(1+k)^{n}]}$$

We can calculate D_{AS}, D_{AL}, D_{LS}, D_{LL} and then CX_{AS}, CX_{AL}, CX_{LS}, CX_{LL} ;

$$D_{AS} = \frac{0.00125(1+0.001652)\left[\frac{1-(1+0.001652)^{-6}}{0.001652}\right]+6(0.001652-0.00125)/(1+0.001652)^{6}}{0.00125+[(0.001652-0.00125)/(1+0.001652)^{6}]}$$

$$= \frac{0.0074691+0.00238823}{0.001648}$$

= 5.98139 months = 0.49845 years

$$D_{AL} = \frac{0.0225(1+0.019804)\left[\frac{1-(1+0.019804)^{-10}}{0.019804}\right]+10(0.019804-0.0225)/(1+0.019804)^{10}}{0.0225+[(0.019804-0.0225)/(1+0.019804)^{10}]}$$

$$= \frac{0.20632224-0.0221591}{0.0202841}$$

= 9.079187 six months = 4.53959 years

$$D_{LS} = \frac{0.000192(1+0.002874)\left[\frac{1-(1+0.002874)^{-6.5}}{0.002874}\right]+6.5(0.002874-0.000192)/(1+0.002874)^{6.5}}{0.000192+[(0.002874-0.000192)/(1+0.002874)^{6.5}]}$$

$$= \frac{0.0012382+0.0171108}{0.0028244}$$

= 6.496601 two weeks = 0.24987 years

$$D_{LL} = \frac{0.005(1+0.003729)\left[\frac{1-(1+0.003729)^{-8}}{0.003729}\right]+8(0.005-0.003729)/(1+0.003729)^{8}}{0.005+[(0.003729-0.005)/(1+0.003729)^{8}]}$$

$$= \frac{0.03948373+0.0098697}{0.0062337}$$

=7 .91797 three months = 1.97930 years

Then, change the k by 1 basis point to calculate convexities,

k_{AS} = 0.02 ± 0.0001= 0.0201/0.0199; monthly k_{AS} = 0.001659764/0.001643397

$$A_{s-} = 3.3479318*[(1-1.001659764^{-6})/0.001659764]$$

$$=3.3479318*5.965299$$

$$=19.97141305$$

$$A_{s+} = 3.3479318*[(1-1.001643397^{-6})/0.001643397]$$

$$= 3.3479318*5.9656393$$

$= 19.97255368$

$CX_{AS} = (19.97141305+19.97255368-2*19.97195412)/19.97195412$

$*(0.0001)^2$

$=292.86$

$k_{AL}= 0.04 \pm 0.0001= 0.0401/0.0399$; six monthly $k_{AS} = 0.019852931/0.019754873$

$A_{L}- = 9.0230146*[(1-1.019852931^{-10})/0.019852931]$

$=9.0230146*8.9895008$

$=81.11239736$

$A_{L+} = 9.0230146*[(1-1.019754873^{-10})/0.019754873]$

$=9.0230146*8.9895008$

$=81.15404301$

$CX_{AL} = (81.11239736+81.15404301-2*81.13317462)/ 81.13317462*$

$(0.0001)^2$

$=112.32$

$k_{LS} = 0.0075 \pm 0.0001 = 0.0076/0.0074$; Bi-weekly $k_{LS} = 0.000291245/0.000283608$

$L_{S}- =1.8474833*[(1-1.000291245^{-6.5})/0.000291245]$

$=1.8474833*6.4929068$

$= 11.9955368$

$L_{S+} = 1.8474833*[(1-1.000283608^{6.5})/0.000283608]$

$=1.8474833*6.5050717$

$=11.99588016$

$CX_{LS} = (11.9955368+11.99588016-2*11.99570967)/ 11.99570967*$

$(0.0001)^2$

$=19.88$

$k_{LL} = 0.015 \pm 0.0001 = 0.0151/0.0149$; Quarterly $k_{LS} = 0.00375381/0.003704366$

$$L_{L-} = 9.970652*[(1-1.00375381^{-8})/0.00375381]$$

$$=9.970652*7.8665999$$

$$=78.4344976$$

$$L_{L+} = 9.970652*[(1-1.003704366^{-8})/0.003704366]$$

$$= 9.970652*7.682729$$

$$= 78.45181069$$

$$CX_{LL} = (78.43512967+78.45181069-2*78.44318425)/ 78.44318425* (0.0001)^2$$

$$= 76.75$$

Now we have D_{AS}, D_{AL}, D_{LS}, D_{LL} and CX_{AS}, CX_{AL}, CX_{LS}, CX_{LL}, we will make some assumptions on the dk of different yield according to real life yield curve:

$$\Delta k_{AS} = 0.003; \Delta k_{AL}= 0.01; \Delta k_{LS} = 0.0001; \Delta k_{LL}= 0.008$$

Based on the equation (7), we can therefore calculate the change in Equity with the new adjusted data:

$$\Delta E = [-(D_{AS}\frac{d_{KAS}}{(1+k_{AS})} - \frac{1}{2}CX_{AS} * d_{KAS}{}^2)A_S$$

$$-(D_{AL}\frac{d_{KAL}}{(1+k_{AL})} - \frac{1}{2}CX_{AL} * d_{KAL}{}^2)\ A_L]-$$

$$[-(D_{LS}\frac{d_{KLS}}{(1+k_{LS})} - \frac{1}{2}CX_{LS} * d_{KLS}{}^2)\ Ls$$

$$-(D_{LL}\frac{d_{KLL}}{(1+k_{LL})} - \frac{1}{2}CX_{LL} * d_{KLL}{}^2)\ L_L\ ]$$

$$\Delta As = [-(D_{AS}\frac{d_{KAS}}{(1+k_{AS})} - \frac{1}{2}CX_{AS} * d_{KAS}{}^2\ )\ ]A_S$$

$$= [-0.49845 \frac{0.003}{(1+0.02)} - \frac{1}{2} 292.86 * (0.003)^2] \; 19.97195412$$

$$= -\; 0.0292795 + 0.0263204$$

$$= -\; 0.0029591$$

$$\Delta A_L = [-(D_{AL} \frac{d_{KAL}}{(1+k_{AL})} - \frac{1}{2} CX_{AL} * d_{KAL}{}^2)] A_L$$

$$= [-4.53959 \frac{0.01}{(1+0.04)} - \frac{1}{2} 112.32 * (0.01)^2] \; 81.13317462$$

$$= -3.5414553 + 0.4556439$$

$$= -\; 3.0858114$$

$$\Delta Ls = [-(D_{LS} \frac{d_{KLS}}{(1+k_{LS})} - \frac{1}{2} CX_{LS} * d_{KLS}{}^2)] L_S$$

$$= [-0.24987 \frac{0.0001}{(1+0.0075)} - \frac{1}{2} 19.88 * (0.0001)^2] \; 11.99570967$$

$$= -\; 0.0002975 + 0.0000012$$

$$= -\; \$\, 0.0002963$$

$$\Delta L_L = [-(D_{LL} \frac{d_{KLL}}{(1+k_{LL})} - \frac{1}{2} CX_{LL} * d_{KLL}{}^2)] L_L$$

$$= [-1.97930 \frac{0.008}{(1+0.015)} - \frac{1}{2} 76.75 * (0.008)^2] \; 78.44318425$$

$$= -\; 1.2237446 + 0.1926565$$

$$= -\; 1.0310881$$

The actual declines in A_S, A_L, L_S and L_L, given annual payment, we can get:

$k_{AS} = 0.02 + 0.003 = 0.023$; monthly $k_{AS} = 0.001896754$

$A_S = 3.3479318*[(1 - 1.001896754^{-6})/0.001896754] = 19.954908$

$\Delta A_s = 19.954908 - 19.97195 = -\; \$\, 0.017042$

$k_{AL} = 0.04 + 0.01 = 0.05$; six monthly $k_{AL} = 0.0246951$

$A_L = 9.0230146*[(1- 1.0246951^{-10})/0.0246951 = 79.094564$

$\Delta A_L = 79.004564 - 81.13317462 = - 2.128611$

$k_{LS} = 0.0075 + 0.0001 = 0.0076$; bi-weekly $k_{LS} = 0.000291245$

$L_S = 1.8474833*[(1-1.000291245^{-6.5})/0.000291245] = 11.99553347$

$\Delta L_S = 11.99553347 - 11.99570967 = - 0.0001762$

$k_{LL} = 0.015 + 0.008 = 0.023$; quarterly $k_{LL} = 0.005701061$

$L_L = 9.970652*[(1-1.005701061^{-8})/0.005701061] = 77.75714398$

$\Delta L_L = 77.75714398 - 78.44318425 = - 0.6860364$

The results derived above are Table 5 below: (Figures are in $ millions)

Table 5

	Actual Decline	The Duration Gap Model: Decline	Convexity Adjusted Duration Gap Model: Decline	Error Reduced:
ΔAs due to 0.3% increase in k_{AS}	–$ 0.017042	–$ 0.0292795	–$ 0.0029591	$ 0.0263204
ΔA_L due to 1% increase in	– $ 2.128611	–$ 3.5414553	–$ 3.0858114	$0.4556439

k_{AL}				
ΔA	**–$ 2.145653**	**–$ 3.5707348**	**–$ 3.0887705**	**$ 0.4819643**
ΔL_S due to 0.01% increase in k_{LS}	– $ 0.0001762	– $ 0.0002975	– $ 0.0002963	$ 0.0000012
ΔL_L due to 0.08% increase in k_{LL}	– $ 0.6860364	– $1.2237446	– $ 1.0310881	$ 0.1926565
ΔL	**–$ 0.6862126**	**–$ 1.2240421**	**–$ 1.0313844**	**$ 0.1926577**
ΔE	**–$ 1.46**	**–$ 2.35**	**–$ 2.06**	**$ 0.29**

Table 5 Convexity Adjusted Duration Gap Model vs Duration Gap Model

As expected, the convexity adjusted model reduces the error of over-predictions of the declines in short-term assets, long-term assets, short-term liabilities, long-term liabilities and equity. Overall, the duration gap model over-predicts the decline by $ 2.35 million while the convexity adjusted duration model over-predicts the decline only by $2.06 million. The error is corrected by the convexity is $0.29 m and this can be significant for a bank which is satisfying the capital adequacy requirements only at margin.

4.0 Some Strategies to Manage Interest Rate Risk in the Context of the Convexity Adjusted Duration Gap Model

4.1 Choice over Durations and Convexities

A bank has some discretion to affect durations of its assets and liabilities by selecting contractual maturities by setting up high or low coupon rates, by choosing fixed or floating credit instruments, by setting up shorter or longer interest rate reset periods for its floating rate credit instruments, by choosing the sizes of prepayment provisions or other inserted options. In the standard duration gap model, a bank with a positive duration gap has some choices to individually affect durations of assets and liabilities, and raise or lower bank's leverage. Keeping everything the same the bank can lower the weighted average duration of assets and/or raise the weighted average duration of its liabilities and/or increase its leverage, the interest rate risk will reduce. In the model of this paper, these strategies continue to exist for the bank in order to manage its interest rate risk. In addition, the model of the study also suggests that the bank can also manage its interest rate risk by affecting the convexities of individual classes of assets and individual classes of liabilities. However, in the strategies based on convexities, the bank has to recognize those factors that affect convexity of a credit instrument, also affect its duration. For example, both duration and convexity vary inversely with coupon rates, with different degrees. However, for the same duration, higher coupon rate debt instruments have higher convexities than lower coupon rate bonds. This analyze suggests that a bank can find combinations of durations and convexities which can lower its interest rate risk.

4.2 The Use of Interest Futures to Hedge Interest Rate Risk

According to Shaffer (1991), traditional measures of interest rate risk are useful but they only provide rough estimates. Therefore, financial derivatives are useful to hedge the interest rate risk. Here, we illustrate the use of the interest rate futures to hedge the decline in the equity of the bank suggested by the application of equation (7) Section 3.

In the previous sub-section, the numerical illustration of equation (7) showed that given expected upward drifts of the yields of short-term assets, long term assets, short-term liabilities and long-term liabilities, the expected loss of the equity of the bank was \$2.08 million. Given the market value of equity of the bank of \$10.67 million before the expected increases in the yields, the decline of 1.46 million represents around 20% which is significant. The bank may like to fully hedge against this loss of its net worth and in order to do so, it needs to take a short position in sufficient number of interest rate futures to create a perfect hedge.

Denoting the bond, underlying the interest rate futures, having the current price of P_b, duration of D_F years and convexity of CX_F, and the current yield on the bond of k_b ; the expected change in the yield of the bond of Δk_b and the current futures price for one contract of F, number of contracts , N_F, which the bank has to sell to have a perfect hedge is given by the following equation:

$$N_F \times \Delta F = (- D_F \frac{\Delta k_b}{(1+k_b)} + \frac{1}{2} CX_F \Delta {k_b}^2) * P_b * N_F \quad (8)$$

If the yields would rise, the short position would provide to the bank with a gain of \$ $N_F \times \Delta F$. In order to create a full hedge, N_F and ΔF have to be equal to ΔE in equation (7), that is

$$N_F \times \Delta F = \left(- D_F \frac{\Delta k_b}{(1+k_b)} + \frac{1}{2} CX_F \, \Delta k_b{}^2 \right) * P_b * N_F$$

$$= \left[- \left(D_{AS} \frac{d_{KAS}}{(1+k_{AS})} - \frac{1}{2} CX_{AS} * d_{KAS}{}^2\right) A_S \right.$$

$$- \left(D_{AL} \frac{d_{KAL}}{(1+k_{AL})} - \frac{1}{2} CX_{AL} * d_{KAL}{}^2\right) A_L \Big] -$$

$$\Big[- \left(D_{LS} \frac{d_{KLS}}{(1+k_{LS})} - \frac{1}{2} CX_{LS} * d_{KLS}{}^2\right) Ls$$

$$\left. - \left(D_{LL} \frac{d_{KLL}}{(1+k_{LL})} - \frac{1}{2} CX_{LL} * d_{KLL}{}^2\right) L_L \right]$$

Suppose the following characteristics of the bond underlying the interest rate futures: the face value, FV = \$1000; the coupon rate, $c = 0.05$; term to maturity, $n = 10$ years and coupon payments are annual. Since $k_b = c$, the bond's current price, P_b , is \$1000.

The duration of the bond, $D_{F,}$ is given by:

$$D_F = \frac{0.05(1.05)\left[\frac{1-1.05^{-10}}{0.05}\right] + \frac{10\,(0.05-0.05)}{1.05^{10}}}{0.05 + \frac{0.05-0.05}{1.05^{10}}} = 8.11 \text{ years}$$

Then we will calculate the convexity, CX_F, we let k_b change by 1 basis point from its current level of 0.05 and get:

$$P_{b+} = 50*[(1-1.0499^{-10})/0.0499] + 1000/1.0499^{10}$$

$$= 1000.772546$$

$$P_{b}- = 50*[(1-1.0501^{-10})/0.0499] + 1000/1.0501^{10}$$

$$= 999.228201$$

$$CX_F = \frac{(1000.772546 + 999.228201 - 2000)}{1000 * 0.0001^2} = 79.7$$

Thus, assuming, $\Delta k_b = 0.01$, ΔF is:

$$\Delta F = \left(-8.11 * \frac{0.01}{1.05} + 0.5 * 79.7 * 0.01^2\right) 1000 = -\$\,73.253$$

Finally, the number of the interest rate futures the banks will need to go short to perfectly hedge against the loss of economic value due to expected upward surge of the yields on bank's assets and liabilities, is:

$$N_F = \frac{2{,}060{,}000}{73.3483} = 28{,}085.177 \text{ contracts}$$

5.0 Conclusions

The Basel Committee (1993) has noted that interest rate risk is a significant impact on the banking industry and also indicated the complexity of getting access to the real-world data. This study noted that despite the fact that the convexity measure reduces the error in the predicted changes in the economic value of a bank in response to changes in interest rates, using the duration gap model, the convexity adjustment is seldom done in the extant literature on interest rate risk of banks. This study proposed a convexity adjustment to the duration gap analysis model to reduce the error of standard framework of measuring interest rate risk.

The study presented the convexity-adjusted duration models in two cases of a hypothetical bank: (i) a single asset and a single liability case and (ii) a two-asset and two-liability case. The model was implemented numerically under simplifying assumptions and showed the reduction in error in the prediction of the decline in equity due to an unexpected increase in interest rates in each of the two cases.

In the Balance Sheet of ScotiaBank (2018), we noted that there were different interest rates for each class of assets and each class of liabilities. This study follows the assumed structure of interest rates as that of ScotiaBank. The assumed interest rates also follow the usual upward sloping yield curve.

There are various ways to hedge interest rate risks such as gap analysis, balance-sheet hedging, scenario analysis and the use of derivatives. In this study, we provide the choices of durations and convexities and also give an example of hedging the interest rate risk by using interest rate future contracts.

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Appendix

Appendix I: Contractual maturities as at October, 31, 2018 (Scotiabank Annual Report, 2018)

T53 Contractual maturities

($ millions)	Less than one month	One to three months	Three to six months	Six to nine months	Nine to twelve months	One to two years	Two to five years	Over five years	No specific maturity	Total
	As at October 31, 2018									
Assets										
Cash and deposits with financial institutions and precious metals	$ 54,254	$ 920	$ 284	$ 101	$ 117	$ 326	$ 726	$ 223	$ 8,509	$ 65,460
Trading assets	4,792	5,311	3,326	5,463	2,309	7,934	12,765	18,130	40,232	100,262
Financial instruments designated at fair value through profit or loss	–	–	–	–	12	–	–	–	–	12
Securities purchased under resale agreement and securities borrowed	74,522	21,223	5,743	673	337	549	539	432	–	104,018
Derivative financial instruments	3,178	5,517	2,024	2,327	1,446	6,447	6,071	10,548	–	37,558
Investment securities – FVOCI	3,925	6,436	5,852	3,284	3,243	13,139	15,206	4,758	1,305	57,148
Investment securities – amortized cost	452	1,429	1,160	1,501	1,500	4,302	9,465	934	–	20,743
Investment securities – FVTPL	–	–	–	–	–	–	–	–	505	505
Loans	40,463	27,581	28,920	27,246	28,064	93,191	214,017	34,985	57,367	551,834
Residential mortgages	11,496	4,697	8,774	12,014	12,781	53,629	126,934	21,366	1,666[1]	253,357
Personal loans	4,204	2,701	3,528	3,431	3,558	11,712	23,338	5,468	38,079	96,019
Credit cards	–	–	–	–	–	–	–	–	16,485	16,485
Business and government	24,763	20,183	16,618	11,801	11,725	27,850	63,745	8,151	6,202[2]	191,038
Allowance for credit losses	–	–	–	–	–	–	–	–	(5,065)	(5,065)
Customers' liabilities under acceptances	13,829	2,082	338	50	30	–	–	–	–	16,329
Other assets	–	–	–	–	–	–	–	–	44,624	44,624
Total assets	195,415	70,499	47,647	40,645	37,058	125,888	258,789	70,010	152,542	998,493
Liabilities and equity										
Deposits	$ 56,965	$53,331	$ 48,661	$ 39,716	$ 32,753	$ 45,262	$ 78,295	$18,313	$303,238	$ 676,534
Personal	8,797	9,415	12,536	9,563	10,241	13,472	11,953	261	138,307	214,545
Non-personal	48,168	43,916	36,125	30,153	22,512	31,790	66,342	18,052	164,931	461,989
Financial instruments designated at fair value through profit or loss	22	77	360	410	523	3,090	1,646	1,969	91	8,188
Acceptances	13,838	2,082	338	50	30	–	–	–	–	16,338
Obligations related to securities sold short	910	972	870	305	1,013	3,896	8,685	7,388	8,048	32,087
Derivative financial instruments	2,520	4,288	1,613	2,716	1,583	6,773	7,699	10,775	–	37,967
Obligations related to securities sold under repurchase agreements and securities lent	96,157	3,466	1,634	–	–	–	–	–	–	101,257
Subordinated debentures	–	–	–	–	–	–	–	5,698	–	5,698
Other liabilities	2,720	592	1,302	422	757	1,784	6,167	5,978	33,022	52,744
Total equity	–	–	–	–	–	–	–	–	67,680	67,680
Total liabilities and equity	173,132	64,808	54,778	43,619	36,659	60,805	102,492	50,121	412,079	998,493
Off-Balance sheet commitments										
Operating leases	$ 36	$ 72	$ 106	$ 104	$ 102	$ 378	$ 818	$ 880	$ –	$ 2,496
Credit commitments[3]	4,232	5,588	13,438	15,182	22,619	23,906	105,988	6,486	–	197,439
Financial guarantees[4]	–	–	–	–	–	–	–	–	36,423	36,423
Outsourcing obligations[5]	18	36	52	52	52	207	311	–	1	729

Figure 1

Appendix II: Contractual maturities as at October, 31, 2017 (Scotiabank Annual Report, 2018)

($ millions)	As at October 31, 2017 Less than one month	One to three months	Three to six months	Six to nine months	Nine to twelve months	One to two years	Two to five years	Over five years	No specific maturity	Total
Assets										
Cash and deposits with financial institutions and precious metals	$ 51,646	$ 894	$ 395	$ 175	$ 159	$ 396	$ 514	$ 290	$ 10,911	$ 65,380
Trading assets	5,484	5,106	3,275	2,740	2,224	5,272	14,816	17,776	41,771	98,464
Financial instruments designated at fair value through profit or loss	–	–	–	–	–	13	–	–	–	13
Securities purchased under resale agreement and securities borrowed	73,346	16,966	3,732	1,087	188	–	–	–	–	95,319
Derivative financial instruments	3,544	4,558	2,084	1,418	1,274	4,303	8,375	9,808	–	35,364
Investment securities – available-for-sale	3,094	5,645	4,495	2,170	2,131	6,506	18,098	7,054	1,311	50,504
Investment securities – held-to-maturity	739	779	1,052	1,193	123	5,847	8,923	109	–	18,765
Loans	28,840	25,032	28,778	29,291	27,197	74,303	209,229	28,667	53,032	504,369
Residential mortgages	3,072	4,065	9,542	15,700	13,083	42,460	129,448	18,017	1,529[1]	236,916
Personal loans	3,980	2,309	3,124	3,322	3,217	10,899	20,601	5,293	36,482	89,227
Credit cards	–	–	–	–	–	–	–	–	14,104	14,104
Business and government	21,788	18,658	16,112	10,269	10,897	20,944	59,180	5,357	5,244[2]	168,449
Allowance for credit losses	–	–	–	–	–	–	–	–	(4,327)	(4,327)
Customers' liabilities under acceptances	10,875	2,399	254	22	10	–	–	–	–	13,560
Other assets	–	–	–	–	–	–	–	–	33,535	33,535
Total assets	177,568	61,379	44,065	38,096	33,306	96,640	259,955	63,704	140,560	915,273
Liabilities and equity										
Deposits	$ 56,154	$ 48,037	$ 49,107	$ 30,938	$ 26,373	$ 44,735	$ 73,099	$ 16,037	$ 280,887	$ 625,367
Personal	7,058	7,247	8,500	7,840	7,862	13,223	13,741	393	134,166	200,030
Non-personal	49,096	40,790	40,607	23,098	18,511	31,512	59,358	15,644	146,721	425,337
Financial instruments designated at fair value through profit or loss	–	3	5	118	133	543	2,882	979	–	4,663
Acceptances	10,875	2,399	254	22	10	–	–	–	–	13,560
Obligations related to securities sold short	336	167	97	148	1,057	3,354	9,229	9,935	6,443	30,766
Derivative financial instruments	2,810	3,348	1,786	1,258	1,347	3,056	11,534	9,061	–	34,200
Obligations related to securities sold under repurchase agreements and securities lent	85,636	8,452	1,524	229	2	–	–	–	–	95,843
Subordinated debentures	–	–	–	–	–	–	–	5,935	–	5,935
Other liabilities	1,419	1,076	440	824	187	1,369	3,223	4,314	30,462	43,314
Total equity	–	–	–	–	–	–	–	–	61,625	61,625
Total liabilities and equity	157,230	63,482	53,213	33,537	29,109	53,057	99,967	46,261	379,417	915,273
Off-Balance sheet commitments										
Operating leases	$ 30	$ 60	$ 88	$ 87	$ 84	$ 311	$ 656	$ 593	$ –	$ 1,909
Credit commitments[3]	4,661	5,913	12,862	18,293	17,254	24,091	97,773	4,819	–	185,666
Financial guarantees[4]	–	–	–	–	–	–	–	–	36,344	36,344
Outsourcing obligations[5]	19	37	54	53	53	207	517	–	1	941

Figure 2

Appendix III: Average Balance Sheet and Net Interest Income in 2016,2017, 2018.

(ScotiaBank, Annual Report, 2018)

T7 Average balance sheet[1] and net interest income

	2018			2017			2016		
For the fiscal years ($ billions)	Average balance	Interest	Average rate	Average balance	Interest	Average rate	Average balance	Interest	Average rate
Assets									
Deposits with financial institutions	$ 54.2	$ 0.9	1.59%	$ 53.2	$ 0.5	0.98%	$ 67.8	$ 0.4	0.58%
Trading assets	101.6	0.2	0.17%	107.2	0.1	0.13%	107.2	0.2	0.16%
Securities purchased under resale agreements and securities borrowed	94.4	0.4	0.47%	97.0	0.3	0.29%	99.8	0.1	0.16%
Investment securities	79.8	1.6	2.01%	74.8	1.3	1.68%	67.8	1.1	1.57%
Loans:									
Residential mortgages	244.2	8.3	3.39%	228.3	7.4	3.23%	218.6	7.4	3.37%
Personal loans	92.1	6.0	6.55%	87.4	5.3	6.08%	84.4	5.0	5.98%
Credit cards	15.1	2.8	18.45%	13.5	2.5	18.73%	12.4	2.3	18.37%
Business and government	177.0	7.9	4.45%	165.0	6.5	3.94%	161.4	5.5	3.41%
Allowance for credit losses	(5.0)			(4.5)			(4.6)		
Total loans	$ 523.4	$ 25.0	4.77%	$ 489.7	$ 21.7	4.43%	$ 472.2	$ 20.2	4.28%
Total earning assets	$ 853.4	$ 28.1	3.29%	$ 821.9	$ 23.9	2.91%	$ 814.8	$ 22.0	2.70%
Customers' liability under acceptances	16.3			12.3			11.4		
Other assets	76.0			78.4			87.6		
Total assets	$ 945.7	$ 28.1	2.97%	$ 912.6	$ 23.9	2.62%	$ 913.8	$ 22.0	2.41%
Liabilities and equity									
Deposits:									
Personal	$ 213.9	$ 3.3	1.52%	$ 203.8	$ 2.7	1.30%	$ 195.1	$ 2.4	1.22%
Business and government	399.8	6.5	1.64%	374.7	4.7	1.26%	384.7	3.9	1.01%
Financial institutions	42.2	0.7	1.77%	42.1	0.5	1.23%	42.8	0.4	1.03%
Total deposits	$ 655.9	$ 10.5	1.61%	$ 620.6	$ 7.9	1.27%	$ 622.6	$ 6.7	1.08%
Obligations related to securities sold under repurchase agreements and securities lent	96.0	0.3	0.25%	102.3	0.2	0.21%	99.1	0.2	0.19%
Subordinated debentures	5.7	0.2	3.71%	7.1	0.2	3.19%	7.5	0.2	3.10%
Other interest-bearing liabilities	60.1	0.9	1.46%	58.5	0.6	0.99%	54.9	0.6	1.04%
Total interest-bearing liabilities	$ 817.7	$ 11.9	1.45%	$ 788.5	$ 8.9	1.13%	$ 784.1	$ 7.7	0.98%
Other liabilities including acceptances	63.9			65.3			74.4		
Equity[2]	64.1			58.8			55.3		
Total liabilities and equity	$ 945.7	$ 11.9	1.26%	$ 912.6	$ 8.9	0.97%	$ 913.8	$ 7.7	0.84%
Net interest income		$ 16.2			$ 15.0			$ 14.3	

Figure 3

Curriculum Vitae

Candidate's full name: Juntao Xu

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Publications: None.

Conference Presentations: No.