Experimental Study of the Flow Induced Impeller Vibrations in a Polymer Mixing Vessel

by

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BEng in Aeronautical, Gujarat Technological University, 2012

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Science in Engineering

In the Graduate Academic Unit of Mechanical Engineering

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This thesis is accepted by the

Dean of Graduate Studies

THE UNIVERSITY OF NEW BRUNSWICK

August, 2016

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Abstract

Vibrational characteristics of a single and double 45° pitched blade impellers (PBIs) operating in a standard baffled mixing vessel were studied. Experiments were conducted for 8 different impeller configurations with combinations of: no of impellers (single or double), diameter to tank diameter ratios $D/D_T = 1/2$ and $5/7$, and impeller spacing of 0, 0.75D, and 1.5D. The power required to drive the impeller, impeller orbit frequency and shape, mean square deflection, and phase relationship were studied with glycerine-water mixture having Reynolds number, $Re \leq 5000$, and $N/N_{crit} \leq 1.2$.

Two high speed cameras were used to measure two orthogonal components of lateral impeller deflection. It was observed that the impeller orbits are periodic at low speed, and exhibit chaotic motion at high speed which can be modelled as a normal distribution. The mean square deflection continues to grow beyond $N^* > 1$ exponentially, and the addition of a second impeller, increases the mean square deflection. Frequencies associated with impeller orbits for the $D/D_T = 1/2$ showed that at low rotational speeds, the highest energy in power spectra of the impeller deflection is at twice the impeller rotational speed. With increase in rotational speed, the energy in the power spectra shifts to synchronous and subsynchronous range with a broad band character. The phase relationship and whirl direction of top and bottom impeller is dependent on the rotational speed and impeller spacing.
To Mom and Dad,
who have always encouraged me to take on new endeavors. Especially this one.
Acknowledgements

I would like to take this opportunity to thank all the individuals whose support has made this work possible.

I am extremely grateful to my supervisor Dr. Holloway. I have become a better researcher and professional as a result of his guidance and willingness to push me towards challenging and significant work.

To the faculty members at the University of New Brunswick, who have been generous in sharing their valuable knowledge and time. A special thanks to Dr. Noel Kippers, former Research Engineer at the UNB Nuclear Group, who gave me much appreciated suggestions from his experiences related to this project.

To the staff at the Department of Mechanical Engineering, who have always been willing to help. Particularly Brian Guidry, who aided in the manufacturing of the experimental apparatus.

To my colleagues and friends who have made this experience really enjoyable. Especially to Kevin Bourque, Arnoldo Paez, and Lance Mills, for countless conversations about work and life over a cup of coffee.

I am extremely indebted to my mother and father, who despite being miles away have always motivated me, been patient and supported my quest for education.

Last but not the least, I would like to thank Natural Sciences and Engineering Research Council of Canada (NSERC) and Nova Chemicals for their financial support.
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<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
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<tr>
<td>$C$</td>
<td>m</td>
<td>Off-bottom clearance</td>
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<td>$C_f$</td>
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<td>Shaft first critical speed</td>
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<tr>
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<td>Unit</td>
<td>Description</td>
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<td>---------------</td>
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<td></td>
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</tr>
<tr>
<td>$O_{ref}$</td>
<td></td>
<td>Center of rotation of the shaft with respect to camera for reference</td>
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<td>Power required</td>
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<td>N m s$^{-1}$</td>
<td>Power required for single impeller</td>
</tr>
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<td>N m s$^{-1}$</td>
<td>Power required for two impellers</td>
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<td>$R_{x,y}$</td>
<td>m$^2$</td>
<td>Autocorrelation</td>
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<tr>
<td>$R_{xy}$</td>
<td>m$^2$</td>
<td>Cross-correlation</td>
</tr>
<tr>
<td>$Re$</td>
<td></td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$S$</td>
<td>m</td>
<td>Spacing between impellers</td>
</tr>
<tr>
<td>$S_k$</td>
<td></td>
<td>Skewness</td>
</tr>
<tr>
<td>$S(\omega)_{x,y}$</td>
<td>[ ]$^2$rad$^{-1}$ s</td>
<td>Power spectral density</td>
</tr>
<tr>
<td>$T$</td>
<td>s</td>
<td>Total time</td>
</tr>
<tr>
<td>$U_{rC}$</td>
<td>m</td>
<td>Whirl amplitude</td>
</tr>
<tr>
<td>$V$</td>
<td>volts</td>
<td>Voltage</td>
</tr>
<tr>
<td>$W$</td>
<td>m</td>
<td>Impeller blade width</td>
</tr>
<tr>
<td>$W_b$</td>
<td>m</td>
<td>Baffle width</td>
</tr>
<tr>
<td>$W_t$</td>
<td>m</td>
<td>Baffle thickness</td>
</tr>
<tr>
<td>$c$</td>
<td>m</td>
<td>Clearance between impeller and baffles</td>
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<tr>
<td>$c_t$</td>
<td>N s m$^{-1}$</td>
<td>Total off diagonal damping</td>
</tr>
<tr>
<td>$c_f$</td>
<td>N s m$^{-1}$</td>
<td>Fluid added off diagonal damping</td>
</tr>
<tr>
<td>$c_s$</td>
<td>N s m$^{-1}$</td>
<td>Structural off diagonal damping</td>
</tr>
<tr>
<td>$e_{per}$</td>
<td>m</td>
<td>Permissible eccentricity</td>
</tr>
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<td>m$^2$</td>
<td>Error due to spatial resolution (for mean square deflection)</td>
</tr>
<tr>
<td>$e_{st}$</td>
<td>m$^2$</td>
<td>Statistical error (for mean square deflection)</td>
</tr>
<tr>
<td>$e_{total}$</td>
<td>m$^2$</td>
<td>Total error (for mean square deflection)</td>
</tr>
<tr>
<td>$e_u$</td>
<td>kg m</td>
<td>Mass eccentricity</td>
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<tr>
<td>$f$</td>
<td>Hz</td>
<td>Frequency</td>
</tr>
<tr>
<td>$f_{mi}$</td>
<td>Hz</td>
<td>Frequency of macro-instability</td>
</tr>
<tr>
<td>$f_{nb}$</td>
<td>Hz</td>
<td>Natural frequency</td>
</tr>
<tr>
<td>$f(x)$</td>
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<td>$g$</td>
<td>m s$^{-2}$</td>
<td>Standard gravity</td>
</tr>
<tr>
<td>$k$</td>
<td>N m$^{-1}$</td>
<td>Total off diagonal stiffness</td>
</tr>
<tr>
<td>$l$</td>
<td>m</td>
<td>Shaft length</td>
</tr>
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### Symbol List

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<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$k_s$</td>
<td>N m$^{-1}$</td>
<td>Shaft stiffness</td>
</tr>
<tr>
<td>$m$</td>
<td>kg</td>
<td>Impeller mass</td>
</tr>
<tr>
<td>$m_n$</td>
<td>kg</td>
<td>Shaft mass (with tungsten)</td>
</tr>
<tr>
<td>$m_{snt}$</td>
<td>kg</td>
<td>Shaft mass (without tungsten)</td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td>Number of samples</td>
</tr>
<tr>
<td>$n_f$</td>
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<td>Fluid refractive index</td>
</tr>
<tr>
<td>$n_t$</td>
<td></td>
<td>Tank refractive index</td>
</tr>
<tr>
<td>$t$</td>
<td>s</td>
<td>Time</td>
</tr>
<tr>
<td>$t_b$</td>
<td>m</td>
<td>Impeller blade thickness</td>
</tr>
<tr>
<td>$x, y$</td>
<td>m</td>
<td>Impeller deflection in the $x$ and $y$-direction</td>
</tr>
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### Greek symbols:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta f$</td>
<td>Hz</td>
<td>Frequency increment</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>s</td>
<td>Time increment</td>
</tr>
<tr>
<td>$\phi$</td>
<td>rad s$^{-1}$</td>
<td>Phase between whirl amplitude and eccentricity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td></td>
<td>Factor for $N_{crit}$ that aligns mean square deflection curve for impeller with different radii</td>
</tr>
<tr>
<td>$\delta$</td>
<td>m</td>
<td>Deflection</td>
</tr>
<tr>
<td>$\delta_s$</td>
<td>m</td>
<td>Structural unbalance amplitude</td>
</tr>
<tr>
<td>$\delta_{total}$</td>
<td>m</td>
<td>Total vibration amplitude</td>
</tr>
<tr>
<td>$\zeta_f$</td>
<td></td>
<td>Non-dimensional fluid viscosity</td>
</tr>
<tr>
<td>$\zeta_s$</td>
<td></td>
<td>Structural damping factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>rad</td>
<td>Angular position</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Pa s</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>[ ]$^n$</td>
<td>Central moment</td>
</tr>
<tr>
<td>$\rho$</td>
<td>kg m$^{-3}$</td>
<td>Fluid density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td></td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$\omega$</td>
<td>rad s$^{-1}$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$\omega_{nb}$</td>
<td>rad s$^{-1}$</td>
<td>Bending natural frequency</td>
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### Superscript:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>Normalized quantity</td>
</tr>
<tr>
<td>\dot{}</td>
<td>First derivative</td>
</tr>
<tr>
<td>\ddot{}</td>
<td>Second derivative</td>
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### Subscript:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>Bottom impeller</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of a series</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial direction</td>
</tr>
<tr>
<td>$t$</td>
<td>Top impeller</td>
</tr>
<tr>
<td>$x$</td>
<td>$x$ direction</td>
</tr>
<tr>
<td>$y$</td>
<td>$y$ direction</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background

Polyethylene is a polymer produced in a continuous process inside stirred tank reactors (mixing vessels), where feedstock and catalyst are mixed at high temperature and high pressure. Fluid in the mixing vessel is moderately shear-thinning (non-Newtonian), with viscosity $\mu$ in the range of 0.024 Pa s - 0.62 Pa s, and density $\rho$ in the range of 910 kg m$^{-3}$ - 965 kg m$^{-3}$ [1]. Mixing within the reactors consists of one or more impellers chosen to achieve the desired degree of mixing [2].

Flow within these reactors have Reynolds numbers in the range of 2000 - 50,000, and it is desired to be turbulent for good mixing [3]. Addition of the radial baffles inside the vessels intensifies the turbulence, and inhibits large scale vortex formation. An undesired consequence of the turbulence is the pulsating loads on mixing impeller which lead to torsional and bending vibrations of the impeller shaft, and the possibility of whirl instability. Excessive vibration of the shaft can lead to fatigue failure, and affect the bearing life adversely. Furthermore, where the clearance between baffles and blades is small, excessive vibration amplitude of the shaft can also damage
the mixing vessel. Mixing vessel components are expensive and such an accident can lead to substantial financial loss in indirect cost such as production loss, contractual penalties, and loss of future orders. The British Hydromechanics Research (BHR) group (formerly known as British Hydromechanics Research Association (BHRA)) investigated such an incident of a 7 m diameter, and 12 m deep mixing vessel, and reported that the direct cost of replacing the mixing shaft, mixing vessel, and loss of a batch of raw material was £80,000, but the indirect costs were over £2 million [4].

It is clear that polymer production requires a high degree of reliability, and minimum uncertainty in the operation of the mixing equipment is desired for the competitive advantage. Existing methods to predict impeller shaft vibration amplitude and frequency are limited to a single impeller only. However, the mixing reactors used in the industry often consist of multiple impellers in a string. Vibrational characteristics of these multiple impellers are significantly different and involve complex fluid flow patterns. Knowledge of the vibrational characteristics of the multiple impeller string is virtually non-existent.

In the present study, the simplified scale model of an industrial polyethylene reactor shown in the figure 1.1 was used. The model reactor has an overhead motor that drives an impeller shaft which is supported with a bushing at the lower end.
1.2 Objectives

The goal of the present study was to extend the knowledge of the vibration characteristics of multiple 45° pitched blade impeller (PBI) strings in a simplified model reactor. The following objectives were set for the study,

(a.) Measurement of the power required to turn single and double PBI strings.

(b.) Measurement of the mean square deflection of single and double 45° PBIs with different radii at different shaft locations and for a range of rotational speeds.

(c.) Measurement of the spectral content of the impeller vibration.

(d.) Measurement of the phase relationship between the motions of two impellers in a string.

Figure 1.1: Simplified drawing of the scale model used in the experiment. Linear dimensions are 4.9:1 to industrial scale.
Chapter 2

Literature Review

Studies conducted on the fluid forces acting on the impeller shaft and flow field in the mixing vessel date from the late 1970s, when an extensive study of the fluid loading on the mixing shaft was carried out by Dr. G.J. Pollard at British Hydromechanics Research Association (BHRA). They found that the mixing impeller shaft loading depends on many factors, including impeller type, rotational speed, baffle configuration, and fluid properties. The objective of the study was to identify which combination of these operating variables gives the lowest deflection levels to serve as design data for the mixing vessels. Their result was published in two proprietary reports of the BHRA [5] [6]. Experiments were performed in a 0.61 m diameter cylindrical mixing tank with dished bottom. A brass mixing shaft having an outer diameter of 62 mm with an impeller in cantilevered configuration was used. A schematic of the experimental apparatus is given in figure 2.1. The following parameters were investigated: rotational speed, fluid height, impeller (type, diameter, no. of blades, and blade width), fluid viscosity, fluid density, baffle number, and tank internals. Measurements were performed using strain gauges mounted on the shaft with 3 output channels (two bending, and one torsion). Results suggested that
bending loads and torque increase smoothly with the rotational speed squared ($N^2$). A summary of the effect of the operating variables on shaft loading studied by Dr. Pollard and his team [5] [6] has been given in table 2.1.

Weetman et. al. (2002) [7] provided guidelines for the mechanical design of the mixing vessels, with focus on the effect of the fluid forces acting on the impeller during its operation. It was observed that the fluctuating fluid forces on the impeller were the result of the transient fluid flow asymmetry around the impeller. Strain gauges were used to obtain the bending loads and torque, which were mounted on the upper part of the shaft. Figure 2.2 shows the fluctuations of bending loads on mixing shaft. Motor current fluctuations were $\pm 5$ to $\pm 15\%$ from the mean value, and bending load fluctuations on the shaft were $\pm 10$ to $\pm 30\%$ from the mean value. Impeller blade loading fluctuations were almost four times compared to what occurs at the motor. Results show the severity of the fluctuations, with torque fluctuations being the smallest of all.
Table 2.1: Summary of the effect of operating variables on the shaft loading studied by Pollard et. al. [5][6].

<table>
<thead>
<tr>
<th>Operating variable</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational speed</td>
<td>Loading increases with $N^2$</td>
</tr>
<tr>
<td>Fluid height</td>
<td>Above $H_f=D/2$ loading is constant</td>
</tr>
<tr>
<td>Impeller type</td>
<td>Flat paddles and wide-bladed impellers had higher loading as compared to pitched blade and narrow-bladed impellers</td>
</tr>
<tr>
<td>No. of baffles</td>
<td>2 or 4 baffles have same loading, but without baffles loading reduces by 50%</td>
</tr>
<tr>
<td>Fluid viscosity</td>
<td>No significant change in damping for small change in viscosity, at highest viscosity $\mu = 1$ Pa s damping was significantly increased</td>
</tr>
<tr>
<td>Shaft eccentricity</td>
<td>No significant change for small eccentricity, but for large value of eccentricity $(D/12)$ loading increases significantly</td>
</tr>
</tbody>
</table>

Laser Doppler Velocimetry (LDV) was also carried out [7], to obtain two components of the velocity fluctuations for the three different impeller types: a Fluidfoil impeller (A310), a pitched blade impeller (A200), and a Rushton impeller (R100). The result is given in the figure 2.3, which shows the dynamic nature of flow near impeller with a high intensity of fluctuations that varies with impeller type. Hence the choice of impeller influences the average load on the blade, as well as the dynamic behavior of the system. The measurements were performed over a range of rotational speeds suggesting that the shaft vibration is maximum at two rotational speeds: a) when rotational speed is the same as the critical speed, and b) when blade passing frequency is coincident with critical speed. Figure 2.4 shows the fluid force measurements over the range of rotational speeds for a A310 impeller (three bladed). It can be seen that there is strong amplification in the fluid force at $N/N_{crit} = 0.33$. This is where the blade passing frequency is coincident with the natural frequency, since it’s a three bladed impeller.
Figure 2.2: Bending load on the mixer shaft ($x$, $y$, and resultant bending) [7].

(a) Main component of velocity.  
(b) Perpendicular component of velocity.

Figure 2.3: Velocity fluctuations measured by LDV [7].

Figure 2.4: Fluid force measurement vs. normalized rotational speed [7].
Kippers et al. (2014) [8] performed an experimental measurement of mean square deflection of a 45° PBI with $D/D_T = 5/7$ in cantilevered configuration using strain gauges and a high speed camera, which traced the orbit of the impeller’s geometric center. A cylindrical tank with diameter $D_T = 0.356\text{ m}$ and fluid height $H_f = 0.356\text{ m}$ was filled using glycerine-water mixture with viscosity ranging from 0.05 Pa s - 0.5 Pa s. Experiments were conducted for the rotational speeds ranging from 148 rpm to 410 rpm (clockwise and anti-clockwise). Results indicated that mean square deflection varied with rotational speed and direction (see figure 2.5). The asymmetry resulted from the combination of the impeller blade pitch, and cantilevered support. At low rotational speeds, the impeller orbits were periodic and as the speed is increased the orbits form complex patterns and then finally become entirely random.

![Figure 2.5: Normalized mean square deflection for four standard PBIs with baffles with compressive and tensile axial loading [9].](image)

Shi et al. (2015) [10] performed experiments and numerical simulations to study the
fluid structure interaction in a mixing tank with a Rushton turbine with $D/D_T = 1/3$, in a standard baffled configuration with tank diameter $D_T = 0.58$ m. The impeller was rotating inside the tank with rotational speeds in the range of $0.4 < N/N_{crit} < 0.6$ in a cantilevered configuration. Bending loads and torque were measured. Measurements were performed using high frequency strain gauges mounted on the shaft. A coupled numerical simulation was performed using commercial codes ABAQUS (handling computational structural dynamics), FLUENT (handling computational fluid dynamics), and MPCCI (handling coupling controls between structure and fluid code). The bending moment and torque obtained experimentally and numerically were in good agreement with 5% relative error in power measurements, and 15% maximum relative error in shaft bending moment coefficient. The PSD of the lateral forces obtained using numerical simulation was analyzed using the Yule-Walker autoregressive model. It was found that the major frequencies involved in the fluctuations in the forces include: a rather low frequency (due to macro-instability in bulk flow), the rotational frequency (presumed due to mass eccentricity), and the blade passing frequency due to lateral forces induced by pseudo-turbulence or vortex behind the blade.

Literature outlined so far was limited only to one impeller, as no study on the fluid forces and vibration for the multiple impellers was found. However, studies on the fluid flow inside tanks with multiple impellers have been done to understand flow phenomenon involved in such mixing vessels.

Armenante et. al. (1996) [11] performed experimental (LDV) and numerical study on the single and double 45° PBIs in 0.29 m diameter mixing tank, filled with water up to height of 0.36 m. Clearance between the bottom of the tank and impeller was $0.33 \ H_f$ for both single, and double impeller case. The spacing between the impellers was $0.33 \ H_f$ for double PBIs case. The measurement of 3 components of fluctuating
velocity were taken at 8 radial locations and 5 different heights. Furthermore, velocity measurements were also performed 5 mm below and above the impeller at 7 locations along impeller radius. Figure 2.6 shows the velocity above the top plane (5 mm), and below the bottom plane (5 mm) for 7 radial locations along the impeller for single and double impeller case. The presence of the top impeller produces a smooth axial profile, due to the homogeneous inflow to the lower impeller. Comparing the two cases (single and double impeller), they inferred that the addition of the second impeller intensifies the axial recirculation loop in central and upper part of the vessel, and also creates a secondary recirculation loop in the bottom part of mixing tank.

Figure 2.6: LDV velocity measurement above and below impeller for single and double PBIs [11].

Hudcova et. al. (1988) [12] studied the power consumption of two Rushton turbines in a 0.56 m diameter tank under aerated and unaerated conditions. Experiments for the single turbine and double turbines were performed with fluid height to tank diameter ratio ($H_f/D_T$) of 1, and 2 respectively. Clearance between turbines was varied from $0.2D$ to $3D$. Experiments were performed in air and water. To measure the shaft torque, two strain gauges were used: 1 fitted between the 2 impellers
(only for the cases where spacing was higher than $D$), and the other above the top impeller. Figure 2.7 shows the variation of the ratio of the power drawn by two versus one impeller $P_2/P_1$ with the impeller spacing. It is evident that the spacing of 2 to 3 impeller diameters is required for maximum power. Figure 2.8 shows the flow patterns as function of impeller spacing $S$.

Figure 2.7: Unaerated power consumption for two impellers compared with single impeller configuration as a function of spacing [12]. $P_2$ denotes power drawn for two impellers, and $P_1$ refers to power drawn with single impeller only.

Paglianti et. al. (2008) [13] studied the features of macro-instability (MI) in stirred tanks with single and multiple impellers. MI in stirred tanks has received a significant attention among all the fluid phenomenon in mixing vessels. MI phenomenon occurs in pseudo-periodic manner at frequencies lower than rotational speed of impeller. For PBIs, the MI frequency can be approximated by the relation $f_{MI} = 0.074N$. A single $45^\circ$ PBI was used for the study, and for the case with multiple impellers,
Rushton and BT-6 type (with 0.151 m and 0.215 m diameter respectively) were used. Experiments were performed in a cylindrical tank with tank diameter $D_T = 0.48$ m. Fluid height to tank diameter ratio $H_f/D_T$ of 1 and 3 was selected for a single impeller and multiple impellers studies, respectively. Schematics of both tanks are given in figure 2.9.

Pressure transducers were attached to the tank wall to obtain time series measurements of pressure, which was sampled at the rate of 500 Hz for an interval of 60 minutes [13]. The results for single impeller MI frequency showed good agreement with the previous data available. For multiple impellers, the results indicated that the MI frequency decreases with the number of impellers. Time series of pressure data from two pressure transducers was cross-correlated to obtain MI velocity. It was found that the MI velocity is proportional to the mean liquid axial velocity in the stirred tank. Results confirmed that two types of MI exist: MI due to jet and MI due to vortex. For a single impeller, the MI due to jet is dominant as compared to MI due to vortex. As the number of impellers increases, MI due to vortex increases.

Figure 2.8: Flow patterns for different impeller spacing. a) $S \ll 0.5D$, b) $0.5 D < S < 1.5D$, and c) $S > 2D$ [12].
Early studies in the field of mixing vessels were focused on the study on the effect of operating conditions on the performance parameters such as power number and mixing time. Recently, the focus has changed towards gaining insight in flow phenomenon in mixing tanks using experimental and numerical techniques, and understanding the effect of operating conditions on the flow field. Even though there is strong interaction between fluid, the mixing shaft, and the impeller structure, this interaction between the fluid and structure inside stirred tanks is not well understood. It is clear that there is gap in the available knowledge of vibrational characteristics of mixing impellers (especially multiple impellers).
Chapter 3

Background Theory

3.1 Standard Mixing Vessel

3.1.1 Tank Geometry

A standard mixing vessel layout has been given in the figure 3.1a. Typically these mixing reactors are vertical cylindrical tanks with diameter $D_T$, having an impeller shaft that is connected to an external motor through a gear-box above the shaft. The tank is filled up to height $H_f$ equal to the tank diameter $D_T$, with the impeller mounted at an off-bottom clearance of $C = H_f/2$. Tanks are equipped with baffles to limit the swirl and large vortices, and also to achieve good mixing. A standard baffled configuration refers to the cylindrical mixing tank with four vertical baffles, each having a width $W_b$ within range of 1/12 - 1/10 of the tank diameter [14], as shown in figure 3.1a. The tanks usually have flat, shallow cone, or ASME dish bottoms; their effect on the flow pattern is essentially equivalent except for solid suspension applications [14]. Most of the mixing vessels have the impeller shaft in
the overhung (cantilevered) configuration [7]. If multiple impellers are attached on the shaft, the nomenclature is the same as the standard tank geometry nomenclature, with the addition of the spacing, $S$, between the impellers as shown in figure 3.1b. The fluid height $H_f$ is typically greater than tank diameter $D_T$.

3.1.2 Impeller Pumping and Power Characteristics

There are two basic types of mixing impellers used in the reactors: a) Axial, and b) Radial. Impellers are produced in a homologous geometrically similar series, with the standard impeller’s design coded as: $A$ refers to an axial type, and $R$ refers to a radial type. In the present experiment, a $45^\circ$ PBI is used, which is an axial-flow impeller (also known as $A - 2$ type impeller). Liquid height of $0.5D - 2D$ can be covered by a single impeller operating in the mixing vessel [14]. Multiple axial impellers are unlikely to produce separate flow patterns (one per impeller) as compared to the
radial impellers. Axial flow impellers can operate in either pumping up or pumping down mode. In pumping down mode, downward flow is generated by the impeller and is converted into upflow which ultimately returns to the impeller through the baffle region [15].

Reynolds Number $Re$, and Power Number $N_p$ play important roles in the scaling of the mixing vessels. They are defined by equation 3.1 and equation 3.2, respectively.

\[ Re = \frac{\rho ND^2}{\mu} \]  

\[ N_p = \frac{P}{\rho D^5 N^3} \]  

The power number is a dimensionless number relating the resistance force to the inertia force, and it is used for estimation of the power consumed for a given impeller. Power number depends on the following parameters: number of impellers and blades, blade angle, tank diameter, location of the impeller, baffle configuration, and Reynolds number. The relationship between power number and Reynolds number for Newtonian fluids is given in figure 3.2. In the laminar range, the power number varies inversely with Reynolds number and approaches an asymptotic value at high Reynolds number (turbulent range). The turbulent range varies for different impeller designs and may begin anywhere between $Re \approx 10^3$ to $Re \approx 10^5$. To define the turbulent range for all impellers the arbitrary value of $Re \geq 10^5$ is used [14]. For non-Newtonian fluids, the relationship between Reynolds number and power number is given by Skelland [16], and is shown in figure 3.3. It is important to note that if the distance between two axial impellers is small, then they behave as a single large impeller, hence pumping capacity and power drawn is reduced on a per impeller basis.
Figure 3.2: Power number and Reynolds number relationship for seven different impellers designs in Newtonian fluid [17].

Figure 3.3: Power number and Reynolds number relationship for various impellers in non-Newtonian fluid [16].
3.2 Rotordynamics

Rotating systems have complex dynamics for which the analytical solutions are only possible for simplified cases [18]. Vibration theory for rotating systems was first developed by August Föppl (Germany) in 1895 and Henry Jeffcott (England) in 1919 [18]. The theory was developed for prediction of vibration for a simple rotor/bearing system, which is known as the Föppl/Jeffcott rotor, or simply the Jeffcott rotor. Figure 3.4 shows a simply supported single mass Jeffcott rotor model with rigid bearings rotating in vacuum.

Figure 3.4: Jeffcott rotor with rigid bearing [18].

The disk has mass $m$ located at the center of a shaft with negligible mass. The geometric center of the disk, and the center of mass of rotor are at $(u_{xC}, u_{yC})$, and $(u_{xG}, u_{yG})$, respectively, with eccentricity $e_u$. The shaft is rotating with rotational speed $\omega$, and the displacement vector $u_C$ with phase angle $\theta$. $\phi$ is the angle between the vectors $u_C$, and $e_u$. To derive the dynamic equations, Newton’s law of motion is applied to the plane rotor disk,

$$m \frac{d^2}{dt^2} (u_{xC} + e_u \cos(\omega t)) = -k_s u_{xC} - c_s \dot{u}_{xC}$$

(3.3a)

$$m \frac{d^2}{dt^2} (u_{yC} + e_u \sin(\omega t)) = -k_s u_{yC} - c_s \dot{u}_{yC}$$

(3.3b)
and,

\[ -mge_x \cos \theta = I_p \ddot{\theta} \quad (3.4) \]

where \( I_p \) is the polar mass moment of inertia of the disc. When the disc is rotating at constant rotating speed (i.e. \( \theta = \omega t \)), the Jeffcott rotor model reduces to two degree of freedom (DOF) rotor model (i.e. only traverse vibration is considered). Neglecting gravity effects the equation 3.3a, and 3.3b reduces to,

\[ m \ddot{u}_x + c_s \dot{u}_x + k_s u_x = m \omega^2 e_x \cos(\omega t) \quad (3.5a) \]

\[ m \ddot{u}_y + c_s \dot{u}_y + k_s u_y = m \omega^2 e_y \sin(\omega t) \quad (3.5b) \]

Equations 3.5a, and 3.5b are uncoupled and can be analyzed independently. Natural frequency \( \omega_{nb} \), and damping \( \zeta_s \) are defined as,

\[ \omega_{nb} = \sqrt{\frac{k_s}{m}} \quad (3.6) \]

\[ \zeta_s = \frac{c_s}{2m \omega_{nb}} \quad (3.7) \]

The whirl radius \( u_{rc} \) is,

\[ u_{rc} = u_x + j u_y \quad (3.8) \]

where \( j = \sqrt{-1} \). Multiplying equation 3.5b by \( j \) and adding to equation 3.5a, gives a complex variable formulation

\[ m \ddot{u}_{rc} + c_s \dot{u}_{rc} + k_s u_{rc} = me_u \omega^2 e^{j\omega t} \quad (3.9) \]

where

\[ e^{j\omega t} = \cos(\omega t) + jsin(\omega t) \quad (3.10) \]
The steady state forced response can be written as,

\[ u_{rC} = U_{rC}e^{j(\omega t - \phi)} \]  \hspace{1cm} (3.11)

where \( U_{rC} \) is the whirl amplitude. Substituting the first, and second derivative of 3.11, into 3.9,

\[ \left\{ (k_s - m\omega^2) + j\omega c_s \right\}U_{rC}e^{-j\phi} = me_u\omega^2 \]  \hspace{1cm} (3.12)

which can be written as,

\[ \left\{ (k_s - m\omega^2) + j\omega c_s \right\}(U_{rC}\cos\phi - jU_{rC}\sin\phi) = me_u\omega^2 \]  \hspace{1cm} (3.13)

Then separating the real and imaginary parts of equation 3.13 gives,

\[ (k_s - m\omega^2)U_{rC}\cos\phi + \omega c_s U_{rC}\sin\phi = me_u\omega^2 \]  \hspace{1cm} (3.14a)

\[ -(k_s - m\omega^2)U_{rC}\sin\phi + \omega c_s U_{rC}\cos\phi = 0 \]  \hspace{1cm} (3.14b)

From equation 3.14b, phase \( \phi \) can be determined as,

\[ \tan\phi = \frac{c_s\omega}{k_s - m\omega^2} \]  \hspace{1cm} (3.15)

Substituting the value of phase \( \phi \) from equation 3.15 into 3.14a, whirl amplitude \( U_{rC} \) can be determined as,

\[ U_{rC} = \frac{m\omega^2e_u}{\sqrt{(k_s - m\omega^2)^2 + (c_s\omega)^2}} \]  \hspace{1cm} (3.16)

in non-dimensional form, equation 3.15 and 3.16 can be written as,

\[ \frac{U_{rC}^*}{e_u} = \frac{\left( \frac{\omega}{\omega_{nb}} \right)^2}{\sqrt{1 - \left( \frac{\omega}{\omega_{nb}} \right)^2 + \left( 2\frac{\omega}{\omega_{nb}} \zeta_s \right)^2}} \]  \hspace{1cm} (3.17)
\[ \phi = \tan^{-1}\left( \frac{2 \frac{\omega}{\omega_{nb}} \zeta \omega}{1 - \left( \frac{\omega}{\omega_{nb}} \right)^2} \right) \]  

(3.18)

Figure 3.5 shows the dimensionless amplitude ratio \( U_{rc}^* \) plotted over the frequency ratio \( \omega/\omega_{nb} \) for different damping ratios. The amplitude peaks at \( \omega/\omega_{nb} = 1 \), and approaches 1 as frequency ratio increases. Furthermore, a high damping ratio reduces amplitude.

![Figure 3.5: Dimensionless amplitude for the Jeffcott rotor vs frequency ratio of the forced response [18].](image)

Due to the assumption that bearings are rigid and symmetric, the disk does not tilt, and the model does not include the gyroscopic effects. Furthermore, no hydrodynamic/aerodynamic or fluid-film cross coupling forces are included in the model.

A linear model for the dynamics of the impeller in the horizontal plane can be described by the following system of equations which includes fluid forces [19][20],

\[
\begin{bmatrix}
M & 0 \\
0 & M 
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix} +
\begin{bmatrix}
C_t & -c_t \\
c_t & C_t
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} +
\begin{bmatrix}
K & -k \\
k & K
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} = 0
\]  

(3.19)
where \( \{x, y\}^T \) represents the vector that includes displacement of the impeller(s) in the horizontal plane. Considering only lateral displacement, the mass, damping, and stiffness matrices can be decomposed as [21],

\[
[M] = \begin{bmatrix}
M_s + M_f & 0 \\
0 & M_s + M_f
\end{bmatrix} \tag{3.20}
\]

\[
[C] = \begin{bmatrix}
C_s + C_f & -c_f \\
c_f & C_s + C_f
\end{bmatrix} \tag{3.21}
\]

\[
[K] = \begin{bmatrix}
K_s + K_f & -k_f - k_s \\
k_f + k_s & K_s + K_f
\end{bmatrix} \tag{3.22}
\]

Fluid forces contribute to both the diagonal and off-diagonal elements of the damping and stiffness matrices, which may be equal or even exceed the structural coefficients in some cases.

### 3.3 Statistical Properties of Impeller Deflection

The lateral components of deflection \( x \), and \( y \) are continuous random variable time series. The probability function \( f(x) \) of a stationary random variable has a cumulative probability \( F(x) \).

\[
f(x) = \frac{d}{dx} F(x) \tag{3.23}
\]

The expected value of the random variable \( x \) is defined as,

\[
E[x] = \int_{-\infty}^{\infty} xf(x)dx \tag{3.24}
\]
The expected value of a sum of random variables is given by,

\[ E[x + y] = E[x] + E[y] \]  \hspace{1cm} (3.25)

The moment of \( n \)th order \( m_n \) and central moment of the \( n \)th order \( \mu_n \) of a random variable \( x \) are defined as,

\[ m_n = \left( x^n \right) \]  \hspace{1cm} (3.26)

\[ = \int_{-\infty}^{\infty} x^n f(x) dx \]  \hspace{1cm} (3.27)

\[ \mu_n = \left( (x - \langle x \rangle)^n \right) \]  \hspace{1cm} (3.28)

\[ = \int_{-\infty}^{\infty} (x - \langle x \rangle)^n f(x) dx \]  \hspace{1cm} (3.29)

Mean square deflection (also referred to as variance) of a random variable \( x \) is defined as,

\[ E[x^2] = \int_{-\infty}^{\infty} (x - \langle x \rangle)^2 f(x) dx \]  \hspace{1cm} (3.30)

Based on equation 3.25 and 3.30, we can conclude that the total mean square deflection for the orbit radius is,

\[ E[r^2] = E[x^2] + E[y^2] \]  \hspace{1cm} (3.31)

Skewness, \( S_k \), and Kurtosis, \( K_u \) of a random variable \( x \) are defined as,

\[ S_k(x) = \frac{\mu_3}{\sigma_x^3} \]  \hspace{1cm} (3.32)

\[ K_u(x) = \frac{\mu_4}{\sigma_x^4} - 3 \]  \hspace{1cm} (3.33)
Skewness is a measure of a symmetry relative to the mean. A zero skewness means the data is symmetric. If skewness is negative, the distribution of the random variable has a longer tail on left side, and if skewness is positive, the distribution curve has a longer tail on right side. Kurtosis is a measure of the degree to which a curve of a probability distribution is peaked or flat-topped. A normal distribution has $K_u = 0$, and periodic distribution has $K_u = -1.5$.

### 3.4 Correlations and Spectra

The autocorrelation of stationary random process $x(\xi, t)$ can be defined as,

$$R_x(\Delta t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t + \Delta t)dt$$  \hspace{1cm} (3.34)

For two stationary random process $x(\xi, t)$, and $y(\xi, t)$, the cross-correlation is defined as,

$$R_{xy}(\Delta t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)y(t + \Delta t)dt$$  \hspace{1cm} (3.35)

The power spectral density (PSD) function is obtained from the Fourier transform of the autocorrelation function,

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\Delta t)e^{-i2\pi f \Delta t}dt$$  \hspace{1cm} (3.36)

where $f$ is the frequency in Hz. Conversely, the autocorrelation can be obtained from the PSD as,

$$R_x(\Delta t) = \int_{-\infty}^{\infty} S_x(f)e^{i2\pi f \Delta t}df$$  \hspace{1cm} (3.37)

Equations 3.34 - 3.37 apply to continuous functions. Experimental data signals are limited to a finite time interval, i.e. $t = 0$, to $t = (n - 1)/F_s$, where $n$ is the total
number of points sampled. The discrete Fourier transform (DFT) can be applied for data sampled at $F_s = 1/\Delta t$, over finite time interval $T = n/\Delta t$, and results in $n$ discrete complex values at frequency $f_k = k/T$. DFT for a real or complex time series $x_i$ is obtained as,

$$
\mathcal{F}_k = \frac{T}{n} \sum_{i=0}^{N-1} x_i e^{-2\pi i \frac{ik}{n}}, \quad k = 0, 1, 2, \ldots, n-1 \tag{3.38}
$$

The first $n/2$ values are independent, and the rest are symmetric about $F_s/2$. The frequency increment in the DFT $\Delta f = 1/T$ [22]. The DFT can be computed in MATLAB with following command:

$$
\mathcal{F}_x = \text{fft}(x) \tag{3.39}
$$

over the frequency limits, $f = 0$ to $F_s/2$ with interval of $F_s/n$. The PSD $S_x(f)$ is computed using the following equation,

$$
S_x(f) = \frac{1}{F_s n} \mathcal{F}_x \text{conj}(\mathcal{F}_x) \tag{3.40}
$$

To autocorrelation is computed as the inverse FFT of the PSD using the following equation,

$$
R_x(\Delta t) = \frac{\text{ifft}(S_x F_s)}{\sigma_x} \tag{3.41}
$$

The cross spectra is computed as equation 3.42

$$
S_{xy}(f) = \frac{1}{F_s n} \mathcal{F}_x \text{conj}(\mathcal{F}_y) \tag{3.42}
$$

A crosscorrelation, associated with two signals is determined using the following equation,

$$
R_{xy}(\Delta t) = \frac{\text{ifft}(S_{xy} F_s)}{\sigma_x \sigma_y} \tag{3.43}
$$
In the case of statistically axisymmetric orbit, we have the following conclusions:

\[ E[x^2] = E[y^2] \]  \hspace{1cm} (3.44)

\[ S_x = S_y \]  \hspace{1cm} (3.45)

\[ R_x = R_y \]  \hspace{1cm} (3.46)

Given these equations, it is possible to improve statistical convergence by combining data in the \( x \) and \( y \) directions.

### 3.5 Dimensional Analysis

The dimensional analysis of the impeller vibration in mixing vessels by Kippers [9] can be applied to the present experiments, with the additional parameter impeller spacing \( S \) for the two impeller system. It has the form,

\[ E[r^*2] = f\left(\zeta_f, N^*, \frac{D}{D_T}, m_i^*, \frac{S}{D}\right) \]  \hspace{1cm} (3.47)

where \( \zeta_f \) is the non-dimensional fluid viscosity is defined as,

\[ \zeta_f = \frac{\mu D}{2\omega_{nb}m_i} \]  \hspace{1cm} (3.48)

Normalized rotational speed \( N^* \), and non-dimensional mass of the impeller \( m_i^* \) can be defined with equations 3.49 and 3.50, respectively.

\[ N^* = \frac{N}{N_{crit}} \]  \hspace{1cm} (3.49)
$m_i^* = \frac{m_i}{\rho D^3}$ \hfill (3.50)

$\zeta_f$ can also be expressed as,

$$\zeta_f = \frac{N^*}{2\pi m_i^* Re}$$ \hfill (3.51)
Chapter 4

Research Methodology

4.1 Experimental Apparatus

4.1.1 Scale Model

All the experiments were conducted in the Fluid Mechanics Laboratory at the University of New Brunswick, Fredericton. The scale model’s geometric parameters and material properties were chosen to achieve geometric and dynamic similarity to the full scale industrial polyethylene reactor. A 4.9:1 scale model of the reactor was designed to perform the experiments (shown in figures 4.1 and 4.2). Mechanical, fluid, and optical properties of the experimental setup are summarized in table 4.1.

The mixing vessel was constructed using clear cast acrylic tubing with an inside diameter of $D_T = 356$ mm and wall thickness of 12.7 mm. The acrylic tubing is fitted between two circular steel end plates. The top end plate is supported with three steel cylindrical rods, which carried the weight of the top plate and transmit load to the bottom plate. The tank had a flat bottom, and its internal height, $H_t =$
793 mm. Four evenly spaced baffles were attached along the tank internal periphery, having width $W_b = 35$ mm and thickness $W_t = 6$ mm. An acrylic lid was provided to prevent vortex formation in the tank during mixing. The circular cross-section of the tank, when filled with fluid, causes a lensing effect. To overcome this, an acrylic square box (enclosure) with refractive index of $n_t = 1.49$ is fitted around the tank. This enclosure was filled with the same fluid as the test section, and provided undistorted optical access. The apparatus is mounted on a heavy base (350 kg) and a set of rubber pads to isolate the apparatus from the structural vibrations.

![Figure 4.1: CAD illustration of the experimental arrangement.](image)

### 4.1.2 Impeller and Shaft Configuration

Two $45^\circ$ PBIs were used in the study, having diameters of 178 mm ($D/D_T = 1/2$), and 254 mm ($D/D_T = 5/7$); as shown in figure 4.3 (A2 type impeller). Impeller properties are summarized in table 4.2. The impeller was attached on an acrylic shaft - partially filled with tungsten powder to obtain dynamic similarity to the full scale model. It
Figure 4.2: Experimental apparatus (not to scale).
was ensured that the center of gravity of the shaft remained unchanged with the addition of tungsten powder. The shaft arrangement is illustrated in figure 4.4. The upper end is supported by conical and cylindrical bearings; the lower end has a Teflon bushing. The spacing between the bushing and shaft is 7 thousandth of an inch (0.18 mm) around periphery. The shaft was driven by 5 and 10 bhp electric motors with variable speed control. Impellers were balanced to Grade 1 to minimize the possibility of deflections due to imbalance.
Figure 4.3: 45° pitched blade impeller (PBI) CAD drawing.

Figure 4.4: Shaft-bearing arrangement.

Table 4.2: Impeller parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Impeller 1</th>
<th>Impeller 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter ((D))</td>
<td>m</td>
<td>0.178</td>
<td>0.254</td>
</tr>
<tr>
<td>Diameter to tank diameter ratio ((D/D_T))</td>
<td>1/2</td>
<td>5/7</td>
<td></td>
</tr>
<tr>
<td>Blade width ((W))</td>
<td>m</td>
<td>0.035</td>
<td>0.050</td>
</tr>
<tr>
<td>Blade thickness ((t_b))</td>
<td>m</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Weight ((m))</td>
<td>kg</td>
<td>1.01</td>
<td>2.1</td>
</tr>
</tbody>
</table>
4.1.3 Model Fluid Properties

In the full scale reactor, the fluid is at 200°C and 21 MPa, and is non-Newtonian [3]. A glycerine and water mixture was chosen as the model fluid in the experiment because it had very similar properties to the polyethylene mixture, with the exception of its Newtonian fluid behavior. Furthermore, glycerine-water mixtures are transparent, providing an easy access for the optical measurements.

In this experiment, glycerine and water were blended to achieve a viscosity of ~ 0.1 Pa·s. When the PBI rotates in fluid, it generates heat and causes a rise in temperature of the fluid. Glycerine-water mixture viscosity is sensitive to temperature changes, so the temperature of the fluid was monitored during the test and was kept at 25±5°C. The enclosure box also acts to maintain the temperature inside the mixing tank. In table 4.3, the glycerine-water mixture properties have been summarized. Values were computed using interpolation from the manual: “Physical Properties of Glycerine and its Solutions” [23]. All the properties are computed for a temperature of 25°C, except the refractive index which is computed for the temperature of 20°C.

<table>
<thead>
<tr>
<th>Viscosity</th>
<th>Glycerin Proportion</th>
<th>Density</th>
<th>Refractive Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa·s</td>
<td>% by Wt.</td>
<td>kg/m³</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>86.0248</td>
<td>1025.25</td>
<td>1.4524</td>
</tr>
</tbody>
</table>

The glycerine-water mixture viscosity was measured using a Brookfield DV-II programmable viscometer before and after each trial. Temperature was measured throughout the experiment using a digital thermometer.
4.2 Shaft Angular Position and Rotational Speed

The primary variables that were monitored in the experiments are: shaft angular position & rotational speed, power drawn by the impeller, and shaft lateral deflection in the horizontal plane. Shaft angular position and rotational speed was monitored using an incremental encoder mounted on the driving shaft. They were recorded for two purposes: to compute the rotational speed of the shaft, and to trigger the camera when the shaft is at $0^\circ$, $90^\circ$, $180^\circ$, and $270^\circ$. The encoder is manufactured by Encoder Products Company (Model 776). It is a quadrature type with channel A, channel B, and index, and has a resolution of 1024 pulses per revolution. The encoder was connected to the computer using Data Acquisition Board NI USB-6211 (National Instruments). The position was read and processed using the LabVIEW software to obtain rotational speed, as well as to trigger the camera using a TTL pulse.

4.3 Power Drawn by Impeller

To experimentally measure the power drawn by the impeller, the current consumed by motor was measured. This method of estimating power is not the most accurate [17], but was selected for the ease of instrumentation. The power consumed by the impeller was computed using the following equation,

$$P = 1.73 \times V \times I \times PF - P_{loss} \quad (4.1)$$

where $P_{loss}$ refers to the mechanical losses in the motor and bearings. The mechanical losses were measured by a no load test, for which the current consumed by motor was
measured with respect to the shaft rotational speed, both with and without impeller and shaft (refer figure A.2 in appendix A). The power factor $PF$, was obtained from the motor performance sheet provided by motor manufacturers of the 5 bhp motor (Appendix A). Due to the unavailability of the performance sheet of the 10 bhp motor, it was assumed that it has same power factor as the 5 bhp motor at the same percent of rated load.

4.4 Impeller Deflection

4.4.1 Camera Setup and Optics

The lateral deflections of the shaft were measured using high speed cameras. Two Photron Fastcam SA3 high speed cameras equipped with macro lens manufactured by Navitar (Model No - Zoom 7000) having a focal length of 18 mm - 108 mm were used. The camera outputs black and white images with the resolution of $1024 \times 1024$ pixels. Images were sampled 4 times per revolution using two cameras (i.e. two orthogonal lateral deflection components), so that low frequency vibrations are resolved. In a limited number of cases, sampling was done at 3000 Hz to allow determination of PSD. Illustration of the camera setup and coordinate system has been given in the figure 4.5.

If the camera is not perpendicular to the surface of the square enclosure, it leads to inaccuracy in the measurement due to parallax. To avoid this, each camera was aligned using a mirror which was placed behind the enclosure surface so that the reflection of the camera can be viewed using the computer monitor. The camera was adjusted until the reflection of lens was a perfect circle, as illustrated in figure 4.6. This procedure was performed each time the camera was moved.
4.4.2 Camera Calibration and Image Processing

Camera calibration refers to the determination of the distance per pixel recorded by the camera, and the center of rotation of the shaft with respect to camera. The camera was calibrated whenever there was a change made to the camera setup or the experimental setup. For calibration, the shaft is rotated by hand and approximately 1000 images were sampled at frequency of 25 Hz. A circular black acrylic disc having diameter of 3 inch was used as a marker. The marker diameter $M_d$ is known. The images are read and processed using the MATLAB Image Processing module. Recorded images are first transformed into complement images, and then to binary images after determination of the appropriate grey threshold, as shown in figure 4.7.

Once the raw image is transformed into a binary image, the location of left ($P_{left}$) and right edge ($P_{right}$) of the marker are identified using MATLAB image processing toolbox, and their difference determines the marker width in pixels $M_d(pixel)$. The center of rotation of the shaft with respect to camera $O_{ref}$, and distance per pixel, $L_{pixel}$, was determined using the equation 4.2, and 4.4 respectively. Figure 4.8 is a graphical representation of the procedure.

$$O_{ref} = \sum_{i=1}^{n} O_{ref_i}$$  \hspace{1cm} (4.2)

where the $O_{ref_i}$ is determined as,

$$O_{ref_i} = \left( P_{left_i} + \frac{P_{right_i} - P_{left_i}}{2} \right)$$  \hspace{1cm} (4.3)

$$L_{pixel} = \frac{M_d}{\sum_{i=1}^{n} M_d_i(pixel)}$$  \hspace{1cm} (4.4)
4.4. Impeller Deflection

Figure 4.5: Schematic drawing of camera setup, and coordinate system. Top view.

Figure 4.6: Camera alignment illustration (inner circle at the radial edge of lens, outer circle at the radial edge of camera mount).

Figure 4.7: Steps for the image processing.

a) Raw Image  →  b) Complement of the Image  →  c) Binary Image
For the single impeller cases, the marker was placed above the impeller; for two impeller cases, markers were placed below for the top impeller, and above for the bottom impeller. The center $O_i$, was then determined for each image, and using the equation 4.5, the deflection was obtained.

$$\delta_i = L_{\text{pixel}}(O_i - O_{\text{ref}})$$  \hspace{1cm} (4.5)

Deflections outside $\delta_i \pm 5\sigma$ were replaced by the average values of the adjacent points (i.e. $\delta_{i+1}$, and $\delta_{i-1}$). To plot the time resolved motion of the deflections at high sampling frequency, the deflections in $x$ and $y$ directions were filtered using a low frequency Butterworth filter for clarity in the time resolved impeller motion. Initial trials confirmed the number of required samples to reach statistical convergence was $n = 680$. Figure 4.9 shows the number of samples required to reach convergence for five trials for the single impeller with $D = 0.254$ m. Furthermore, to determine if sampling rate of $4N$ was adequate, mean square deflection for the case with impeller with $D = 0.178$ m and spacing of $0.75D$ was compared with the mean square deflection determined using sampling frequency $F_s = 3000$ Hz, as shown in figure 4.10, and 4.11.

Error bars indicate the sum of two types of errors: a) Statistical error $e_{\text{stat}}$, and b) Error due to spatial resolution $e_{\text{spa}}$. Statistical error is obtained by computing standard deviation of the running variance $E[r^2]$ as shown in figure 4.9. The error due to spatial resolution of the optical measurement was obtained as the square of the minimum pixel resolution (distance per pixel) $L^2_{\text{pixel}}$. The total error $e_{\text{total}}$ was computed as equation 4.6.

$$e_{\text{total}} = e_{\text{stat}} + e_{\text{spa}}$$  \hspace{1cm} (4.6)
4.4. Impeller Deflection

Figure 4.8: $O_{ref_i}$ vs $n$; dashed line represents averaged $O_{ref}$ over $n$ samples.

Figure 4.9: The statistical convergence of $E[r^2]$ for five trials for the single impeller with $D = 0.254$ m.
4.4. Impeller Deflection

Figure 4.10: Normalized mean square deflection for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; top impeller. Symbols correspond to: $\bigcirc$, $F_s = 4N$; $\bullet F_s = 3000$ Hz.

Figure 4.11: Normalized mean square deflection for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; bottom impeller. Symbols correspond to: $\bigcirc$, $F_s = 4N$; $\bullet F_s = 3000$ Hz.
Chapter 5

Results

5.1 Test Conditions

In the present study, the effect of the rotational speed, impeller size, position, and spacing on the impeller vibration were evaluated. Test conditions are summarized in table 5.1.

Two high speed cameras were used to measure the two lateral components of impeller deflection in the horizontal plane. From the images gathered, the mean square response, impeller orbit, and Probability Density Function (PDF) were computed for all the geometric configurations. Typically, 680 images were sampled at the sampling rate of $4 \times$ rotational speed, except for where PSD, correlation and autocorrelation were required. In these cases, the sampling rate was 3000 Hz for high temporal resolution. Power Spectral Density (PSD), autocorrelation, and cross-correlation were computed only for the single and double $D= 0.178$ m impellers. In the case of the single impeller trials, the impellers were secured at a height of $H_f/2$ from the tank bottom. In the case of two impellers, the impellers were secured equidistant
from mid-height on the shaft with a spacing of 0, 0.75\(D\), and 1.5\(D\), and with their blades aligned vertically (i.e. no phase). Rotational speed was varied from zero up to the maximum obtainable speed with the available motor drive and maximum allowable deflection without damage.

Table 5.1: Case matrix.

<table>
<thead>
<tr>
<th>#Impellers</th>
<th>Spacing</th>
<th>a) (D = 0.178) m</th>
<th>b) (D = 0.254) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>-</td>
<td>rpm 50 - 1154</td>
<td>50 - 700</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(N^*) 0.05 - 1.21</td>
<td>0.05 - 1.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Re) 270.70 - 6247.79</td>
<td>551.21 - 7716.96</td>
</tr>
<tr>
<td>Touching</td>
<td>0.75(D)</td>
<td>rpm 50 - 852</td>
<td>50 - 450</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(N^*) 0.05 - 1.21</td>
<td>0.05 - 0.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Re) 270.70 - 4612.75</td>
<td>551.21 - 4960.90</td>
</tr>
<tr>
<td>Double</td>
<td>1.5(D)</td>
<td>rpm 50 - 965</td>
<td>50 - 374</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(N^*) 0.05 - 1.34</td>
<td>0.05 - 0.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Re) 270.70 - 5224.54</td>
<td>551.21 - 4123.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rpm 50 - 963</td>
<td>50 - 381</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(N^*) 0.05 - 1.21</td>
<td>0.05 - 0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Re) 270.70 - 5213.71</td>
<td>551.21 - 4200.22</td>
</tr>
</tbody>
</table>

5.2 Shaft Natural Frequency

The natural frequency of the shaft-impeller-support assembly plays an important role in scaling of the predicted shaft vibrations through the normalization of the rotational speed. Natural frequency varies for different locations of the impeller position, impeller properties, number of impellers, and support arrangement. Submerging the shaft and impeller into liquid also alters the critical speed due to the added fluid mass, damping, and stiffness to be considered.

The natural frequencies of the rotating assembly in vacuum for each configuration were obtained using the rotordynamic code developed by Friswell et. al. [24]. The
MATLAB code uses a finite element approach, where the shaft consists of 10 nodes and 9 Timoshenko beam elements. Shear, rotary inertia, and gyroscopic effects were included in the model. Shaft supports were modelled as pinned, and fixed-pinned. A schematic of the rotordynamic model for the two impeller system is shown in figure 5.1. Mode shapes for the single impeller system and two impeller system at the first four natural frequency are shown in figures 5.2-5.5. Frequencies associated with the first four normal modes are shown in table 5.2.

Table 5.2: Natural frequency calculated using the rotordynamic code by Friswell et. al. [24]. FW refers to a forward whirl, and BW refers to a backward whirl.

<table>
<thead>
<tr>
<th>Impeller</th>
<th>Spacing</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BW</td>
<td>FW</td>
<td>BW</td>
<td>FW</td>
</tr>
<tr>
<td>Pinned - Pinned Support</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impeller 1</td>
<td>Touching</td>
<td>11.69</td>
<td>11.73</td>
<td>93.79</td>
<td>103.69</td>
</tr>
<tr>
<td>(D = 0.178\ \text{m})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75D</td>
<td>11.95</td>
<td>12.02</td>
<td>71.48</td>
<td>75.37</td>
<td></td>
</tr>
<tr>
<td>1.5D</td>
<td>13.23</td>
<td>13.44</td>
<td>49.48</td>
<td>49.98</td>
<td></td>
</tr>
<tr>
<td>Touching</td>
<td>11.39</td>
<td>11.45</td>
<td>124.85</td>
<td>158.59</td>
<td></td>
</tr>
<tr>
<td>0.75D</td>
<td>8.79</td>
<td>8.93</td>
<td>41.35</td>
<td>43.40</td>
<td></td>
</tr>
<tr>
<td>1.5D</td>
<td>10.79</td>
<td>11.39</td>
<td>32.77</td>
<td>32.96</td>
<td></td>
</tr>
<tr>
<td>Fixed - Pinned Support</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impeller 1</td>
<td>Touching</td>
<td>17.59</td>
<td>17.68</td>
<td>105.62</td>
<td>115.37</td>
</tr>
<tr>
<td>(D = 0.178\ \text{m})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75D</td>
<td>18.08</td>
<td>18.22</td>
<td>80.71</td>
<td>84.31</td>
<td></td>
</tr>
<tr>
<td>1.5D</td>
<td>20.38</td>
<td>20.69</td>
<td>58.63</td>
<td>59.06</td>
<td></td>
</tr>
<tr>
<td>Touching</td>
<td>16.51</td>
<td>16.54</td>
<td>147.79</td>
<td>194.95</td>
<td></td>
</tr>
<tr>
<td>0.75D</td>
<td>13.04</td>
<td>13.26</td>
<td>46.45</td>
<td>48.17</td>
<td></td>
</tr>
<tr>
<td>1.5D</td>
<td>7.16</td>
<td>18.1</td>
<td>38.51</td>
<td>38.57</td>
<td></td>
</tr>
</tbody>
</table>
5.2. Shaft Natural Frequency

Figure 5.1: Schematic of the two impeller system with \( D = 0.254 \) m, and spacing of 0.75\( D \); pinned-pinned support.

Figure 5.2: Orbit shapes for the single impeller system with \( D = 0.254 \) m; pinned-pinned support. Symbols correspond to: \( \times \), start of orbit; \( \diamond \), end of orbit. Shaft rotating anticlockwise.

Figure 5.3: Impeller orbits and mode shapes for the single impeller system in vacuum with \( D = 0.254 \) m; pinned-pinned support.
The natural frequency of the rotating assembly for the single impeller with $D = 0.254$ m, was measured with an accelerometer using a bump test in air. Acceleration was recorded for the bump test for five trials. The frequency with the highest energy in the power spectral density was identified as the first natural frequency. The measured frequency for the single impeller with $D = 0.254$ m was 11.28 Hz (average of 5 trials). The corresponding natural frequency obtained using the rotordynamic code was within 0.97% and 1.49% for backward whirl and forward whirl, respectively.

If we observe the schematic of the shaft-bearing arrangement in figure 4.4 closely, the system has the possibility of transitioning from the pinned-pinned condition to the
fixed-pinned condition. At low speed, the system can be assumed to be in pinned-
pinned support condition. At high speeds, the axial loads on the impeller (tensile)
are large and this may push the top bearing inside the bearing hub, making it stiffer,
so that it approximates a fixed-pinned condition. The difference between the natural
frequencies for these two support system is very significant as reported in table 5.2.

5.3 Power Number

The power number for the range of Reynolds number tested is shown in figure 5.6
for all the impeller configurations. In the case of two impellers, the power number
refers to the power consumed per impeller. Results for the lowest rotational speeds
are not shown because the power consumption was too low to measure accurately.

For the small impellers ($D = 0.178$ m), the power number rises with increasing
Reynolds number ($Re > 3500$) until the flow inside the tank becomes turbulent,
after which the power number becomes asymptotically constant. Power number
for multiple impeller becomes constant at lower Reynolds number than the single
impeller. In the case of the large impellers ($D = 0.254$ m), the power number is
nearly constant in the range of $1500 < Re < 3500$, after which the power number
rises rapidly. This is thought to be due to the lateral loads of the impeller, causing
significant friction between shaft and the bottom bushing. The power number values
for $Re > 3500$ for $D = 0.178$ m impeller, and $Re < 3500$ for $D = 0.254$ m impeller
in table 5.3, which summarizes power number, $N_P$, and its uncertainty, $\sigma_{N_P}$.
Figure 5.6: Power number per impeller vs Reynolds number. Symbols correspond to, for $D = 0.178$ m impeller: <, impeller at center; □, two impeller system with $1.5D$ spacing; ⊿, two impeller system with $0.75D$ spacing; ▽, two impeller system with $0$ spacing (touching). For $D = 0.254$ m impeller: ▶, impeller at center; ○, two impeller system with $1.5D$ spacing; ◊, two impeller system with $0.75D$ spacing; △, two impeller system with $0$ spacing (touching).

Table 5.3: Typical power number values in the constant range.

<table>
<thead>
<tr>
<th>Impeller</th>
<th>Spacing</th>
<th>$N_P$</th>
<th>$\sigma_{N_P}$</th>
<th>$P_2/P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = 0.178$ m</td>
<td>-</td>
<td>1.2</td>
<td>0.32</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Touching</td>
<td>0.5</td>
<td>0.24</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>0.75D</td>
<td>0.7</td>
<td>0.20</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>1.5D</td>
<td>0.7</td>
<td>0.15</td>
<td>1.5</td>
</tr>
<tr>
<td>$D = 0.254$ m</td>
<td>-</td>
<td>2.0</td>
<td>0.26</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Touching</td>
<td>1.3</td>
<td>0.10</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>0.75D</td>
<td>1.2</td>
<td>0.23</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>1.5D</td>
<td>1.6</td>
<td>0.28</td>
<td>1.5</td>
</tr>
</tbody>
</table>
5.4 Impeller Orbits

Impeller orbits were determined from the instantaneous deflection measurements sampled at 4 times the rotational speed. Typical results are shown in figure 5.7 and 5.8 for 170 revolutions for the single impeller case with $D = 0.178$ m and $D = 0.254$ m, respectively. It can be observed that, as the rotational speed increases, the orbits become larger and more complex. At low speeds, the orbits are periodic and become increasingly random at high speeds. To assess the statistical symmetry of the deflections in $x$ and $y$ directions, the mean square deflections were plotted for all the trials in figure 5.9. From this, it can be inferred that the motion is statistically axisymmetric and that $E[x^*]^2 \approx E[y^*]^2$.

Typical Probability Density Function (PDF) of the normalized deflections, for the single impeller case with $D = 0.178$ m, and $D = 0.254$ m has been shown in figure 5.10, and 5.11 respectively. Due to axisymmetry, the PDF has been computed using a combined time series of $x^*$, and $y^*$ denoted as $[x^*; y^*]$. The PDF of a normally distributed random variable and periodic variable have been shown for comparison. It can be seen that at low speed, the PDF is close to the periodic motion and tends towards a random normal distribution at high speed. Figures 5.12 and 5.13 show the PDF of normalized deflections at $N^* \approx 0.6$ for the two impeller configurations with $D = 0.178$ m and $D = 0.254$ m impeller, respectively. We can observe that for $D = 0.178$ m impeller, the PDF of the two impellers is very similar for all the spacing. However, for the $D = 0.254$ m impeller, only the $1.5D$ spacing PDFs are similar. For the decreased spacing, the top and bottom impeller have different motions.
Figure 5.7: Impeller orbits for the single impeller with $D = 0.178$ m.
Figure 5.8: Impeller orbits for the single impeller with $D = 0.254$ m.
Figure 5.9: $E[x^2]$, and $E[y^2]$ calculated for all trials.
Figure 5.10: Histogram of normalized deflections, for the single impeller with $D = 0.178$ m.
5.4. Impeller Orbits

Figure 5.11: Histogram of normalized deflections, for the single impeller with $D = 0.254$ m.
Figure 5.12: Histogram of normalized deflections at $N^* \approx 0.6$, for the two impeller system with $D = 0.178$ m. Top : touching, Middle: spacing 0.75$D$, Bottom: spacing 1.5$D$. Left : top impeller, Right: bottom impeller.
Figure 5.13: Histogram of normalized deflections at \( N^* \approx 0.6 \), for the two impeller system with \( D = 0.254 \text{ m} \). Top: touching, Middle: spacing 0.75\( D \), Bottom: spacing 1.5\( D \). Left: top impeller, Right: bottom impeller.
The kurtosis of the instantaneous deflections in $x$ and $y$ direction versus normalized rotational speed for all configurations is shown in figure 5.14. Generally the kurtosis transitions from a value of -1.5 corresponding to periodic motion at low speed, to a value of 0 corresponding to a normal distribution at high speed.

![Figure 5.14: Kurtosis of the normalized deflection in $x$ direction. Symbols correspond to, for $D = 0.178$ m impeller: $\bullet$, impeller at center; $\blacksquare/\square$, two impeller system with $1.5D$ spacing; $\blacklozenge/\diamondsuit$, two impeller system with $0.75D$ spacing; $\blacktriangledown/\triangledown$, two impeller system with 0 spacing (touching). For $D = 0.254$ m impeller: $\triangledown$, impeller at center; $\blackcirc/\bigcirc$, two impeller system with $1.5D$ spacing; $\star/\bigstar$, two impeller system with $0.75D$ spacing; $\blacktriangle/\triangle$, two impeller system with 0 spacing (touching). Hollow symbols represent top impeller, and filled symbols represent bottom impeller. The solid line represents the kurtosis of a normal distribution, while the dashed line represents the kurtosis of a periodic variable.](image)

To observe the time resolved motion of the impeller, deflections for the single impeller with $D = 0.178$ m for 5 rotations were sampled at $F_s = 3000$ Hz, and these are shown in figure 5.15. At the low speed, the orbits are circular but, as the speed increases, the orbit radius increases gradually and circular orbits turn into elliptical orbits, followed by chaotic orbits. Impeller orbits and PDF for the other cases have been plotted in figure B.1 - B.24 in appendix B.
Figure 5.15: Orbit trace for 5 rotations at $F_s = 3000$ Hz for the single impeller with $D = 0.178$ m.
5.5 Mean Square Deflection as a Function of Rotational Speed

The mean square radial impeller deflection $E[r^2]$ was obtained using,

$$E[r^2] = E[x^2] + E[y^2]$$ (5.1)

The mean square deflection of the single $D = 0.178$ m and $D = 0.254$ m impellers are shown in figures 5.16 and 5.17, respectively.

The mean square deflection of the top impeller, the bottom impeller, and their comparison for the two impeller system with $D = 0.178$ m and $D = 0.254$ m have been plotted in figures 5.18 - 5.19, respectively. They demonstrate similar characteristics with deflection of the larger impeller reaching larger values.

Figures 5.18 and 5.19 show that the deflections for the cases of two impellers. The top, and bottom impeller mean square deflection are very similar, except for the case with $D = 0.178$ m, and spacing of $1.5D$. In this case, the mean square deflection of the bottom impeller is larger than the top impeller. This phenomenon could not be confirmed for the $D = 0.254$ m with spacing of $1.5D$, because of limitation of power and structural strength of the apparatus.
5.5. Mean Square Deflection as a Function of Rotational Speed

Figure 5.16: Normalized mean square deflection of the single impeller with $D = 0.178$ m.

Figure 5.17: Normalized mean square deflection of the single impeller with $D = 0.254$ m.
Figure 5.18: Normalized mean square deflection for the two impeller system with $D = 0.178$ m. Top: touching, Middle: spacing 0.75$D$, Bottom: spacing 1.5$D$. Symbols correspond to: ○, top impeller; ●, bottom impeller.
Figure 5.19: Normalized mean square deflection for the two impeller system with $D = 0.254$ m. Top: touching, Middle: spacing $0.75D$, bottom: spacing $1.5D$. Symbols correspond to: $\circ$, top impeller; $\bullet$, bottom Impeller.
5.6 Frequency Analysis and Autocorrelations

Frequency analysis was based on deflection data obtained for the small impeller ($D = 0.178 \text{ m}$) at a constant sampling rate, $F_s = 3000 \text{ Hz}$. Instantaneous deflection data was transformed into equal sized blocks, with each block containing data of 10 shaft rotations. The power spectral density (PSD) was calculated for each block and then averaged over all blocks. The PSD of the deflections were further averaged for the $x$ and $y$ direction based on axisymmetry. The frequency resolution was determined for the record length of each block, $n$, which varied with the rotational speed as, $T = 10 \text{ rev}/N$. PSD for the single impeller with $D = 0.178 \text{ m}$ has been shown in figure 5.20.

![Figure 5.20: PSD of normalized deflections for the single impeller with $D = 0.178 \text{ m}$.

$N^* = 0.17$

$N^* = 0.47$

$N^* = 0.62$

$N^* = 1.07$]
PSD for the two impeller configurations are shown in figures 5.21 - 5.26. For most of the configurations, the highest energy in the power spectrum is associated with normalized frequency $\omega^* = 2$, which is equal to twice the rotational speed. The exception was the PSD for the two impeller system with $D = 0.178$ m and spacing of $0.75D$ for top impeller, as shown in figure 5.23. Here, the most significant energy in the power spectrum is associated with normalized frequency $\omega^* = 1$ (synchronous motion). Beyond $N^* = 1$, the subsynchronous energy becomes most significant with a broad band character.

Figure 5.21: PSD of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of 0 (touching); top impeller.
Figure 5.22: PSD of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of 0 (touching); bottom impeller.
Figure 5.23: PSD of normalized deflection for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; top impeller.
5.6. Frequency Analysis and Autocorrelations

Figure 5.24: PSD of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; bottom impeller.
Figure 5.25: PSD of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; top impeller.
Figure 5.26: PSD of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; bottom impeller.
Autocorrelations of the normalized deflections for the single impeller with $D = 0.178$ m were obtained from the corresponding PSD and are shown in figure 5.27. It can be observed that at low speeds the signal is periodic, and as the rotational speed increases the signal becomes more random. A periodic motion with the same frequency as shaft rotational frequency would have an autocorrelation value of -1 at $180^\circ$. The autocorrelation in figure 5.27 has the lowest value at $90^\circ$, which means the orbit whirling frequency is twice the rotational frequency (i.e. supersynchronous). This is consistent with the corresponding PSD which has energy peak at $\omega^* = 2$.

Figure 5.27: Autocorrelation of normalized deflection, for the single impeller with $D = 0.178$ m. Speeds shown are: $N^* = 0.17$; $N^* = 0.47$; $N^* = 0.62$; $N^* = 1.07$.

Autocorrelations of the two impeller system with spacing of 0 are shown in figures 5.28, and 5.29 for the top, and bottom impeller respectively. The autocorrelation for the top impeller is similar to the autocorrelation for the single impeller case; with supersynchronous whirl at low speed, gradually becoming random. The bottom impeller shown in figure 5.29 shows strong harmonics before becoming random.
Autocorrelations of the top and bottom impeller for the two impeller system with 0.75\(D\) spacing are shown in figures 5.30 and 5.31, respectively. The bottom impeller shows a supersynchronous whirl at low speed, gradually becoming random. For the top impeller, it can be inferred that the whirl is synchronous at low speeds and it becomes random at high speed. The autocorrelation of the top and bottom impellers for the two impeller system with 1.5\(D\) spacing are shown in figures 5.32 and 5.33, respectively. They are similar to the autocorrelation for the single impeller case; with supersynchronous whirl at low speed.
5.6. Frequency Analysis and Autocorrelations

Figure 5.28: Autocorrelation of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of 0 (touching); top impeller. Speeds shown are: \[ N^* = 0.22; \quad N^* = 0.63; \quad N^* = 1.03; \quad N^* = 1.24. \]

Figure 5.29: Autocorrelation of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of 0 (touching); bottom impeller. Speeds shown are: \[ N^* = 0.22; \quad N^* = 0.63; \quad N^* = 1.03; \quad N^* = 1.24. \]
Figure 5.30: Autocorrelation of normalized deflections for the two impeller system with \( D = 0.178 \) m, and spacing of \( 0.75D \); top impeller. Speeds shown are:\n
- \( N^* = 0.22 \);
- \( N^* = 0.62 \);
- \( N^* = 1.00 \);
- \( N^* = 1.35 \).

Figure 5.31: Autocorrelation of normalized deflections for the two impeller system with \( D = 0.178 \) m, and spacing of \( 0.75D \); bottom impeller. Speeds shown are:\n
- \( N^* = 0.22 \);
- \( N^* = 0.62 \);
- \( N^* = 1.00 \);
- \( N^* = 1.35 \).
5.6. Frequency Analysis and Autocorrelations

Figure 5.32: Autocorrelation of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; top impeller. Speeds shown are: ---, $N^* = 0.20$; -- -- -- , $N^* = 0.56$; ---- - - , $N^* = 0.90$; ······· , $N^* = 1.22$.

Figure 5.33: Autocorrelation of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; bottom impeller. Speeds shown are: --- , $N^* = 0.20$; -- -- -- , $N^* = 0.56$; ---- - - , $N^* = 0.91$; ······· , $N^* = 1.22$. 
5.7 Correlated Motion between Impellers

To study the phase relationship between the top and bottom impellers, the four camera arrangements shown in figure 5.34 were used. Camera arrangements (b) and (c) allow study of the phase relationship between $x$ and $y$ deflections of top and bottom impellers, respectively. Camera arrangement (a) allows study of the relationship between top and bottom impeller deflection in either the $x$ or $y$ directions respectively, and camera arrangement (d) allows study of the relationship of the top and bottom deflection in either the $x$ or the $y$ directions, respectively.

![Figure 5.34](image)

Figure 5.34: Schematic diagram of the camera arrangement for two impellers study.

The cross-correlation of the $x$ and $y$ deflections for the single impeller with $D = 0.178$ m is shown in figure 5.35. For most speeds, the cross-correlation has a peak at $45^\circ$, which identifies it as a supersynchronous whirl at twice the shaft rotational speed. At high speed, the $x$ and $y$ deflections are weakly correlated, and the motion is random with a small periodic component.
5.7. Correlated Motion between Impellers

Figure 5.35: Cross-correlation of normalized deflections in \( x \) & \( y \) direction, for the single impeller with \( D = 0.178 \) m. Speeds shown are: \( N^* = 0.17 \), \( N^* = 0.47 \), \( N^* = 0.62 \), \( N^* = 1.07 \).

5.7.1 Camera Arrangement - A

Phase relationships between deflections of the top impeller, \( x_t \), and bottom impeller, \( y_b \), were studied with camera arrangement (a). Cross-correlation for an impeller spacing of 0 is given in figure 5.36. At low speed, the phase lag between \( x_t \) and \( y_b \) is approximately \( 90^\circ \). The difference between consecutive correlation peaks with rotational speed has constant value of \( \sim 180^\circ \), but the phase between \( x_t \) and \( y_b \) changes with rotational speed. This suggests that the whirl frequencies remain unchanged but there is a phase lag between impellers which is a function of rotational speed. Another important observation is that the top, and bottom impellers are anticorrelated, meaning they whirl in opposite directions.

For the spacing of \( 0.75D \) (shown in figure 5.37), the cross-correlation is mostly random with a periodic component with varying phase lag. At low speeds, the top and
bottom impeller are correlated and have the same whirl direction, which becomes anticorrelated with increase in speed, suggesting a change in whirl direction.

The cross-correlation for the spacing of $1.5D$ is given in figure 5.38, which is similar to the cross-correlation for the single $D = 0.178$ m impeller; $x_t$ and $y_b$ are well correlated at low speeds with a phase lag of $\sim 45^\circ$. Phase between $x_t$ and $y_b$ have very small changes slowly with rotational speed as compared to the impeller spacing of 0 and $0.75D$.

![Figure 5.36: Cross-correlation of normalized deflections in $x_t$ & $y_b$ direction, for the two impeller system with $D = 0.178$ m, and spacing of 0. Speeds shown are: ---, $N^* = 0.22$; ---, $N^* = 0.63$; ---, $N^* = 1.03$; ........, $N^* = 1.24$. Camera arrangement (a).]
Figure 5.37: Cross-correlation of normalized deflections in \( x_t \) & \( y_b \) direction, for the two impeller system with \( D = 0.178 \) m, and spacing of 0.75\( D \). Speeds shown are: 

- - - , \( N^* = 0.22 \);
- - - - , \( N^* = 0.62 \);
- - - - , \( N^* = 1.01 \);
- - - - - - - - , \( N^* = 1.36 \).

Camera arrangement (a).

Figure 5.38: Cross-correlation of normalized deflections in \( x_t \) & \( y_b \) direction, for the two impeller system with \( D = 0.178 \) m, and spacing of 1.5\( D \). Speeds shown are: 

- - - - - - - - , \( N^* = 0.20 \);
- - - - , \( N^* = 0.56 \);
- - - - , \( N^* = 0.91 \);
- - - - - - - - , \( N^* = 1.22 \).

Camera arrangement (a).
5.7.2 Camera Arrangement - B

The phase relationship between deflections for top impeller, \( x_t \), and \( y_t \) was studied with camera arrangement (b). For the spacing of 0 and 1.5\( D \), the cross-correlation (shown in figures 5.39, and 5.41 respectively). The phase characteristics are similar to the single \( D = 0.178 \) m impeller. The difference between consecutive correlation peaks is constant at approximately 180°, and the phase difference between motion is 45°, which identifies a supersynchronous whirl. As speed increases, the cross-correlation becomes weaker for both 0 and 1.5\( D \) spacing. The cross-correlation for the spacing of 0.75\( D \) is shown in figure 5.40 with a peak at 90°, meaning a synchronous whirl. As speed increases, the cross-correlation become very weak.

![Cross-correlation graph](image)

Figure 5.39: Cross-correlation of normalized deflections in \( x_t \) & \( y_t \) direction, for the two impellers system with \( D = 0.178 \) m, and spacing of 0. Speeds shown are: \( \cdots \cdots \cdots \), \( N^* = 0.22 \); \( \cdots \cdots \cdots \), \( N^* = 0.63 \); \( \cdots \cdots \cdots \), \( N^* = 1.03 \); \( \cdots \cdots \cdots \), \( N^* = 1.24 \). Camera arrangement (b).
Figure 5.40: Cross-correlation of normalized deflections in $x_t$ & $y_t$ direction, for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$. Speeds shown are: \( \bullet \), $N^* = 0.22$; \(- - - -\), $N^* = 0.62$; \(- - - -\), $N^* = 1.00$; \( \cdots \cdots \cdots \), $N^* = 1.35$. Camera arrangement (b).

Figure 5.41: Cross-correlation of normalized deflections in $x_t$ & $y_t$ direction, for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$. Speeds shown are: \( \bullet \), $N^* = 0.20$; \(- - - -\), $N^* = 0.56$; \(- - - -\), $N^* = 0.90$; \( \cdots \cdots \cdots \), $N^* = 1.22$. Camera arrangement (b).
5.7.3 Camera Arrangement - C

The phase relationships between deflections of bottom impeller $x_b$ and $y_b$ were studied with camera arrangement (c). The cross-correlation for the spacing of 0 is given in the figure 5.42, which shows that the motion of the bottom impeller is periodic with strong harmonics at low speeds. With increase in speed, the motion becomes random quite rapidly. For the spacing of $0.75D$ and $1.5D$, the cross-correlation results are shown in figures 5.43, and 5.44, both showing similar phase characteristics as the single $D = 0.178$ m impeller with a supersynchronous whirl and constant phase lag of $45^\circ$, except for the high speed case, $N^* = 1.35$ in figure 5.43.

![Figure 5.42: Cross-correlation of normalized deflections in $x_b$ & $y_b$ direction, for the two impeller system with $D = 0.178$ m, and spacing of 0. Speeds shown are: ——, $N^* = 0.22$; ——, $N^* = 0.63$; ——, $N^* = 1.03$; ——, $N^* = 1.24$. Camera arrangement (c).](image)
Figure 5.43: Cross-correlation of normalized deflections in $x_b$ & $y_b$ direction, for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$. Speeds shown are: \[ \cdash , \ N^* = 0.22; \cdash \cdash \cdash , \ N^* = 0.62; \cdash \cdash \cdash \cdash , \ N^* = 1.00; \cdash \cdash \cdash \cdash \cdash \cdash , \ N^* = 1.35. \] Camera arrangement (c).

Figure 5.44: Cross-correlation of normalized deflections in $x_b$ & $y_b$ direction, for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$. Speeds shown are: \[ \cdash , \ N^* = 0.20; \cdash \cdash \cdash , \ N^* = 0.56; \cdash \cdash \cdash \cdash , \ N^* = 0.91; \cdash \cdash \cdash \cdash \cdash \cdash , \ N^* = 1.22. \] Camera arrangement (c).
5.7.4 Camera Arrangement - D

The phase relationship between deflection for top impeller, $x_t$, and for bottom impeller, $x_b$ was studied with camera arrangement (d). For the spacing of 0, the cross-correlation is shown in 5.45. It can be observed that the phase angle is a strong function of the rotational speed.

![Figure 5.45: Cross-correlation of normalized deflections in $x_t$ & $x_b$ direction, for the two impeller system with $D = 0.178$ m, and spacing of 0. Speeds shown are: $N^* = 0.22$; $N^* = 0.63$; $N^* = 1.03$; $N^* = 1.24$. Camera arrangement (d).](image)

The cross-correlation for the spacing of 0.75$D$ is given in figure 5.46. At low speed, $x_t$ and $x_b$ are correlated, and with increase in speed, the $x_t$ and $x_b$ become anticorrelated suggesting a shift from having the same whirl directions to the opposite.

For the spacing of 1.5$D$, the cross-correlation show the top and bottom impeller are in-phase at low speeds as shown in figure 5.47. With increase in speed, the phase has very small change which can be considered to be in-phase.
5.7. Correlated Motion between Impellers

Figure 5.46: Cross-correlation of normalized deflections in $x_t$ & $x_b$ direction, for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$. Speeds shown are: \(\cdots, N^* = 0.22; \quad \cdots, N^* = 0.62; \quad \cdots, N^* = 1.00; \quad \cdots, N^* = 1.35\). Camera arrangement (d).

Figure 5.47: Cross-correlation of normalized deflections in $x_t$ & $x_b$ direction, for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$. Speeds shown are: \(\cdots, N^* = 0.20; \quad \cdots, N^* = 0.56; \quad \cdots, N^* = 0.91; \quad \cdots, N^* = 1.22\). Camera arrangement (d).
Chapter 6

Discussion

6.1 Power Number

The power number, $N_P$, for the single $D = 0.178$ m impeller approaches an asymptotic constant value for $Re = 6000$. The result for the $D = 0.254$ m impeller is quite different, remaining constant in the range of $Re = 3000-3500$ and increasing sharply with increased speed. Addition of a second impeller causes the $N_P$ to become constant earlier than the single impeller. As shown in figure 5.6, the power number increases as the blade-baffle clearance decreases for both single and double impellers, which agrees with current literature (Kippers [8], and Chappele [25]). As the spacing between impellers is reduced, the power ratio, $P_2/P_1$ decreases, suggesting that the impellers tend to behave as a single impeller with double blade height, which reduces the overall power requirement for a two impeller system. For the range of Reynolds number where the power number was constant, the maximum power ratio, $P_2/P_1$ is 1.5 for both the $D = 0.178$ m, and $D = 0.254$ m impellers. This finding agrees with current literature by Hudcova et. al. [12], which concluded that, for impeller spacing $S < 2D$, impellers do not draw full power ($P_2/P_1 < 2$). Variation of the
power number, $N_P$, and power ratio, $P_2/P_1$, with the spacing between impellers is due to the bulk flow patterns [12]. Hudcova et. al. [12] identified three zones with respect to flow patterns as summarized in table 6.1 [12]. They found that, for two Rushton turbine with the spacing of $1.5D$, the $P_2/P_1 = 1.91$, whereas, in our study $P_2/P_1$, for the spacing of $1.5D$ was 1.5.

Table 6.1: Flow zones characterized in terms of impeller spacing [12].

<table>
<thead>
<tr>
<th>Spacing</th>
<th>Flow Pattern</th>
<th>$P_2/P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S &lt; 0.5D$</td>
<td>Similar as single larger impeller</td>
<td>30 % increase as compared to single impeller</td>
</tr>
<tr>
<td>$0.5 &lt; S &lt; 1.5D$</td>
<td>One circulation loop per impeller with fluid rotating between them</td>
<td>Rate increase is small</td>
</tr>
<tr>
<td>$S &gt; 2D$</td>
<td>Individual flow pattern</td>
<td>$P_2/P_1 &gt; 2$</td>
</tr>
</tbody>
</table>

6.2 Frequency and Direction of Impeller Orbits

The power spectral density showed that, at low speeds, there is substantial energy in the supersynchronous range ($\omega^* = 2$), and as the speed increases, the energy in the synchronous and subsynchronous range gradually becomes dominant.

To verify the existence of supersynchronous whirl and the whirl direction, the orbits were traced for half shaft rotation for the single impeller with $D = 0.178$ m at low, moderate, and high speeds in figure 6.1. At low speed, it can be observed that, for half a shaft rotation, the impeller orbit whirl is a complete circle. At moderate speed, the orbit is still circular and complete, whereas, at high speed, the orbit shows no identifiable shape. We can therefore conclude that the whirl orbit is in-fact forward and supersynchronous to the shaft rotation.
6.3 Scaling the Radial Deflection

Weetman et al. (2002) reported that the shaft vibration is maximum at two frequencies: a) when rotational speed is the same as natural frequency, and b) when the blade passing frequency is coincident with critical speed [7]. In the present results, a small hump was observed at the $N^* = 0.9$ and 0.7 for the single $D=0.178$ m and 0.254 m impeller respectively, but no peaks were observed at the blade passing frequency (i.e. $N^*/4$, for a four bladed impeller).

In the high speed regime, $N^* > 1$, the mean square deflection is seen to increase exponentially, similar to the finding by Kippers for tensile axial loading [8]. Submerging the shaft and impeller into glycerine-water mixtures reduces the natural frequency due to added mass and damping of the fluid. In order to align the local maximum in the $E[r^2]$ curve for the single impeller with $D = 0.174$ m and $D = 0.254$ m, a factor $\alpha$ of 0.9 and 0.7 (for small and large impellers, respectively) was obtained through trial and error for $N_{crit}$ in air such that $N^*_l = N/N_{crit-l} \approx 1$ as shown in figure 6.2. Once aligned at their peak, the curves are consistent over most of the speed range.
6.4 Correlation Between Impeller Motions

Figure 6.2: Comparison of the total normalized mean square deflection $E[r^2_*]$ for single impellers only. Symbols correspond to: ▲, $D = 0.254$ m; ◼, $D = 0.178$ m.

Using the same factors of $\alpha$, the natural frequencies for the two impeller system was adjusted, and the $E[r^2_*]$ curves for the $D = 0.178$ m and $D = 0.254$ m are shown in figure 6.3. The factor aligns the $E[r^2_*]$ curves for two impeller system with spacing of 0, but the $E[r^2_*]$ curves for two impeller system with $1.5D$, and $0.75D$ do not fully align. This may be due to the difference in added mass and damping for the two impeller system with $1.5D$, and $0.75D$ as compared to the single impeller system.

6.4 Correlation Between Impeller Motions

The finite element rotordynamic analysis of the two impeller system in vacuum presented in section 5.2 suggests that the first two modes of vibration are whirls with the two impellers rotating in the same direction, either backward or forward. The 3rd and 4th modes have the impellers whirling in opposite directions and at
Figure 6.3: Comparison of the total normalized mean square deflection $E[r^*]^2$ for two impeller system. Top: touching, Middle: spacing 0.75$D$, Bottom: spacing 1.5$D$. Symbols correspond to: $>/<$, $D = 0.254$ m; $>/<$, $D = 0.178$ m. Hollow symbols represent top impeller, and filled symbols represent bottom impeller.
considerably higher frequency.

Some of the notable features of the measured cross-correlations for the two impeller system were:

1. At 1.5\(D\) spacing, the cross-correlation between top and bottom impeller suggests that both impellers have whirls in same direction with no significant phase difference, suggesting that the vibration is at 1st or 2nd mode.

2. For the spacing of 0.75\(D\), the impellers have whirl in the same direction at low speed with vibration at lower modes. With increase in speed, the impeller whirl direction becomes opposite, suggesting a shift from lower vibration modes to higher vibration modes, with the phase being a function of rotational speed.

3. Cross-correlation results for the 0 spacing show that the top impeller and bottom impeller are anticorrelated, suggesting that the whirls have opposite directions. This indicates that the vibration is of a higher mode associated with a high frequency. This finding further explains the reason for strong harmonics in the bottom impeller autocorrelation and cross-correlation as discussed earlier. The phase between top and bottom impeller whirl varies significantly with the rotational speed.

### 6.5 Full Scale Deflection at High Speed

The radial mean square deflection for all cases considered in this study has been plotted as a function of rotational speed in figure 6.4. In the range of \(N^* > 1.1\), the mean square deflection increases exponentially with normalized rotational speed.
with an upper limit to the amplitude that can be described using,

\[ E[r^2] = 5.62 \times 10^{-7} \times e^{4N^*} \]  

Figure 6.4: Normalized mean square radial deflection for all single and double impeller cases. Symbols correspond to, for \( D = 0.178 \) m impeller: ▼, impeller at center; □/◊, two impeller system with \( 1.5D \) spacing; ◇/◇, two impeller system with \( 0.75D \) spacing; ▽/▽, two impeller system with 0 spacing (touching). For \( D = 0.254 \) m impeller: △, impeller at center; ○/○, two impeller system with \( 1.5D \) spacing; ★/★, two impeller system with \( 0.75D \) spacing; ▲/▲, two impeller system with 0 spacing (touching). Hollow symbols represent top impeller, and filled symbols represent bottom impeller. Data for the two impeller system with \( D = 0.178 \) m, and spacing of \( 0.75D \) bottom impeller has been removed from the plot. Solid line corresponds to the equation 6.1

In dimensional terms, the mean square radial deflection for a mixer with impeller
diameter, \( D \), turning at speed, \( N \), can be obtained as,

\[
E[r^2] = D^2 \times 5.62 \times 10^{-7} \times \frac{e^{4N/N_{crit}}}{5.62 \times 10^{-7} \times e^{4N/N_{crit}}}
\] (6.2)

The present experiments were designed to minimize the effect of structural imbalance so equation 6.2 only provides an estimate of the fluid contribution to the total deflection. To obtain the total vibration amplitude we assume that the deflection due to imbalance and that due to fluid forces are uncorrelated,

\[
\delta_{total} = \sqrt{\delta_s^2 + E[r^2]}
\] (6.3)

The unbalance amplitude \( \delta_s \) can be obtained as [20],

\[
\delta_s = \frac{m e_u \omega^2}{\sqrt{(k_s - m \omega^2)^2 + ((c_s + c_f) \omega)^2}}
\] (6.4)

where eccentricity \( e_u \) is defined as,

\[
e_u = \frac{G \times 1000}{\omega}
\] (6.5)

where \( G \) is the grade of balance (ISO 1940/1) in unit of mm/s. The fluid damping \( c_f \) can be computed using the equation developed by Cudmore et. al. [21]

\[
c_f = 0.1 \times \rho D^3 2\pi N
\] (6.6)

For the full scale standard rector with parameters given in table 6.2, \( \delta_{total} \), \( \delta_s \), and fluid amplitude \( \sqrt{E[r^2]} \) are compared for a grade G200 imbalance in figure 6.5. It can be observed that even with such high unbalance the \( \delta_s \) is significantly smaller than the fluid contribution, \( \sqrt{E[r^2]} \) in the high speed regime.
Figure 6.5: Deflection amplitude comparison for a hypothetical full scale standard reactor. Amplitudes shown are: \( \delta_{\text{total}} \), \( \delta_s \), \( \sqrt{E[r^2]} \).

### 6.6 Blade-Baffle Impact

The probability of an impact of the blade with a baffle due to excessive deflection, \( r \), for a bivariate normal distribution was given by Kippers [8] as,

\[
P[r \geq c] = e^{-c^2} E[r^2]
\]

(6.7)

Table 6.2: Hypothetical full scale standard reactor parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impeller diameter ((D))</td>
<td>m</td>
<td>1.24</td>
</tr>
<tr>
<td>Impeller mass ((m))</td>
<td>kg</td>
<td>500</td>
</tr>
<tr>
<td>Shaft bending stiffness ((k_s))</td>
<td>N m(^{-1})</td>
<td>9.6 x 10(^6)</td>
</tr>
<tr>
<td>Structural damping factor ((\zeta_s))</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>Critical speed ((N_{\text{crit}}))</td>
<td>rpm</td>
<td>1323.2</td>
</tr>
<tr>
<td>Support configuration</td>
<td></td>
<td>Same as experiment</td>
</tr>
</tbody>
</table>
where $c$ is the clearance between impeller and baffles as,

$$
c = \frac{D_T - D - W_b}{2}
$$

(6.8)

Using equation 6.1, the mean square deflection, $E[r^2]$, can be used to determine the probability of an impact for a given rotational speed. Table 6.3 and figure 6.6 show the computed probability of an impact over a range of normalized rotational speeds $N^* = 0 - 4$ for a baffle with width $W_b = D_T/10$, and $D/D_T = 1/2$, and $5/7$.

Table 6.3: Probability of an impact at rotational speed $N^*$, for a baffle width, $W_b = D_T/10$.

<table>
<thead>
<tr>
<th>Probability</th>
<th>$N^*$ ($D/D_T = 1/2$)</th>
<th>$N^*$ ($D/D_T = 5/7$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1e^{-6}$</td>
<td>2.35</td>
<td>1.56</td>
</tr>
<tr>
<td>$1e^{-5}$</td>
<td>2.39</td>
<td>1.60</td>
</tr>
<tr>
<td>$1e^{-4}$</td>
<td>2.45</td>
<td>1.66</td>
</tr>
<tr>
<td>$1e^{-3}$</td>
<td>2.52</td>
<td>1.73</td>
</tr>
<tr>
<td>$1e^{-2}$</td>
<td>2.62</td>
<td>1.83</td>
</tr>
<tr>
<td>$1e^{-1}$</td>
<td>2.79</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Figure 6.6: Probability of blade impacting the baffle with width, $W_b = D_T/10$. Lines represent: ———, $D/D_T = 1/2$ impeller; ---, $D/D_T = 5/7$ impeller.
Chapter 7

Summary and Conclusions

The experimental study of the flow induced impeller deflections in a mixing vessel with single and double 45° pitched blade impellers was carried out. The effect of impeller diameter, location/spacing of the impellers, and number of impellers was studied over a range of rotational speeds. The impeller shaft was supported by a roller bearing on one end and a bushing on the other. The power required to drive the impeller, impeller orbit structure, mean square deflection, frequency spectra, and phase relationship for 8 different impeller configurations in a glycerine-water mixture were measured. The goal of the study was to extend the knowledge of vibrational characteristics of multiple pitched blade impellers in a baffled mixing vessel.

Specific conclusions of the study were:

1. Impeller orbits are periodic at low speed, and as the speed increases, the orbit becomes larger. At high speed, the orbits are random, and the deflections are normally distributed. Impeller orbits are statistically axisymmetric over the entire range of operating speeds.
2. The mean square deflection $E[r^*r^*]$ continues to grow beyond $N^* > 1$ exponentially as, $E[r^*r^*] \propto e^{4N^*}$. Based on the assumption that the impeller deflection has a bivariate normal distribution, the probability of blade-baffle impact was estimated.

3. For subsynchronous turning rates the highest energy in the power spectra of the impeller deflection is associated with $\omega^* = 2$, which is twice the rotational speed. At high speed, the power spectra has most of its energy in the synchronous and subsynchronous range.

4. Addition of a second impeller increases the mean square deflection at a given rotational speed. The phase between top and bottom impeller whirl is a function of impeller spacing and rotational speed. Reduction in impeller spacing, increases the tendency of impellers whirling in opposite directions, suggesting an impeller vibrating at higher mode.

7.1 Recommendations

1. The knowledge of the phase relationship between multiple impellers can be further investigated. The phase relationship between impellers was studied for small impeller ($D= 0.178$ m) only and the phase relationship between impellers with different $D/D_T$ ratio can be different. This will require substantially higher motor power.

2. The study was limited to two impellers only with a maximum spacing of $1.5D$. This can be further extended for more than 2 impellers. The relationship between longer impeller strings might have complex vibrational characteristics as compared to two impellers.
References


Appendix A

Motor Performance Curves
Figure A.1: Motor performance curve.
Figure A.2: No load test of current versus rotational speed. Symbols correspond to: ●/○, 10 bhp motor; ▲/▶, 5 bhp motor. Hollow symbols represent no load test performed with motor only, and filled symbol represents the no load test performed with motor and shaft including bottom bushing but without liquid in the tank.
Appendix B

Additional Plots - Impeller Orbits
Figure B.1: Impeller orbits for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; top impeller.
Figure B.2: Impeller orbits for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; bottom impeller.
Figure B.3: Histogram of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; top impeller.
Figure B.4: Histogram of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $1.5D$; bottom impeller.
Figure B.5: Impeller orbits for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; top impeller.
Figure B.6: Impeller orbits for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; bottom impeller.
Figure B.7: Histogram of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; top impeller.
Figure B.8: Histogram of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of $0.75D$; bottom impeller.
Figure B.9: Impeller orbits for the two impeller system with $D = 0.178$ m, and spacing of 0 (touching); top impeller.
Figure B.10: Impeller orbits for the two impeller system with $D = 0.178\ m$, and spacing of 0 (touching); bottom impeller.
Figure B.11: Histogram of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of 0 (touching); top impeller.
Figure B.12: Histogram of normalized deflections for the two impeller system with $D = 0.178$ m, and spacing of 0 (touching); bottom impeller.
Figure B.13: Impeller orbits for the two impeller system with $D = 0.254$ m, and spacing of $1.5D$; top impeller.
Figure B.14: Impeller orbits for the two impeller system with $D = 0.254$ m, and spacing of $1.5D$; bottom impeller.
Figure B.15: Histogram of normalized deflections for the two impeller system with $D = 0.254$ m, and spacing of $1.5D$; top impeller.
Figure B.16: Histogram of normalized deflections for the two impeller system with $D = 0.254$ m, and spacing of $1.5D$; bottom impeller.
Figure B.17: Impeller orbits for the two impeller system with $D = 0.254$ m, and spacing of $0.75D$; top impeller.
Figure B.18: Impeller orbits for the two impeller system with $D = 0.254$ m, and spacing of $0.75D$; bottom impeller.
Figure B.19: Histogram of normalized deflections for the two impeller system with $D = 0.254$ m, and spacing of $0.75D$; top impeller.
Figure B.20: Histogram of normalized deflections for the two impeller system with \( D = 0.254 \) m, and spacing of \( 0.75D \); bottom impeller.
Figure B.21: Impeller orbits for the two impeller system with $D = 0.254$ m, and spacing of 0 (touching); top impeller.
Figure B.22: Impeller orbits for the two impeller system with $D = 0.254$ m, and spacing of 0 (touching); bottom impeller.
Figure B.23: Histogram of normalized deflections for the two impeller system with $D = 0.254$ m, and spacing of 0 (touching); top impeller.
Figure B.24: Histogram of normalized deflections for the two impeller system with $D = 0.254$ m, and spacing of 0 (touching); bottom impeller.
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Raval, S., Thakkar, D., Unsteady Aerodynamic Characteristics of Dragonfly in Slow
Speed Climb, CSME International Congress 2014, Toronto, June 2014