The Effects of Gravity on Droplet Transport in Grid Turbulence

by

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Abstract

Droplets in the desired range of 10-140 µm were introduced into grid-generated turbulence using a spray nozzle. Droplet velocity and flux were measured downstream with Phase Doppler Interferometry to study the effects of gravity on droplet migration. The PDI measurement produced two coincident droplet velocity components as well as droplet diameters. This provided the basis for calculation of a concentration field, two-component velocity correlation fields and droplet velocity-concentration field correlations.

The downwards migration of large droplets was observed with downstream distance. The droplets with $d < 60 \mu m$ demonstrated a sink velocity equivalent to that of 60 µm droplet in still air whereas the sink rate of droplets $> 60 \mu m$ was substantially larger.
Acknowledgements

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Nomenclature

$C_D$ Drag Coefficient of Droplet
$E[ ]$ Expected value or mean value
$f_d$ Probability density function of droplet diameter
$f_{v_i}$ Probability density function of droplet velocity in $i$-direction
$f$ Frequency
$Re_d$ Reynolds number of droplet
$St$ Stokes number
$X$ Droplet position (m)
$x$ Instantaneous droplet position (m)
$X$ Mean droplet position (m)
$U$ Gas phase velocity (m/s)
$V$ Droplet phase velocity (m/s)
$C_v$ Volume concentration ($m^3/m^3$)
$C_{v_0}$ Centerpoint volume concentration ($m^3/m^3$)
$c_v$ Instantaneous volume concentration ($m^3/m^3$)
$C$ Droplet number concentration (#/$m^3$)
$g$ Gravitational acceleration ($m/s^2$)
$u$ Fluctuating component of gas phase velocity (m/s)
$u_i'$ Root-mean squared fluctuating carrier phase velocity in $i$-direction (m/s)
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Subscripts

\(c\) Carrier/continuous phase
\(d\) Dispersed phase
\(e\) Energy-containing eddies
\(k\) Kolmogorov scales
\(x\) Horizontal direction
\(y\) Vertical direction
\(z\) Streamwise direction
\(s\) Small droplets: \(d < 25 \mu m\)
\(m\) Medium droplets: \(25 \leq d < 40 \mu m\)
\(l\) Large droplets: \(40 \leq d < 60 \mu m\)
\(xl\) Extra Large droplets: \(d \geq 60 \mu m\)
Chapter 1

Introduction

1.1 Motivation

Polydisperse spray plumes are a common feature of industrial and agricultural applications. A polydisperse spray is considered in which the dispersed phase is a dilute mixture within the carrier phase with a range of droplet sizes. For the present study, droplet sizes and flow conditions were chosen to correspond with significant gravitational effects: droplet size range, $15\mu m < d < 250\mu m$ corresponding to a Stokes number range, $0.049 < St_k < 13.7$. The model flow chosen for this study was a point source in grid turbulence.

An immense amount of work has been applied to produce numerical and analytical models of two phase droplet transport (see Elghobashi, 1994; Moreau et al., 2009; Ryan et al., 2016). Of particular interest are large-eddy simulations models (LES) (Fox, 2012) due to their computational efficiency, but these methods require models of subgrid droplet transport which in atmospheric conditions cannot be resolved below the characteristic length, $\Delta$, on the order of 1 m due to dynamic coupling of varying length scales. The present study has been designed to provide data that will form a basis for modelling this subgrid droplet transport.
1.2 Literature Review

There is a large amount of theoretical literature available on two-phase flow (Bourgoin and Xu, 2014; Mashayek and Pandya, 2003; Monchaux et al., 2012; Tenneti and Subramaniam, 2014; Toschi and Bodenschatz, 2009; Balachander, 2009; Peirano and Minier, 2002). However, in the present study we restrict the discussion to cases where the droplets are small enough to be spherical (due to surface tension), but large enough to have significant inertial effects. In which case the droplets may be considered equivalent to inertial particles.

Experimental research of particle transport in approximately homogeneous turbulence (Bateson and Aliseda, 2012; Bourgoin et al., 2011; Joly et al., 2012; Nicolai et al., 2011; Salazar et al., 2008; Saw et al., 2008; Xu and Bodenschatz, 2008) have provided a baseline for criticism of numerical and theoretical results in terms of particle size, turbulence effects, and droplet interaction. However, the important issue for practical applications regarding the overall transport of particles and spatial gradients of the volume concentration field are not addressed in these studies.

The droplet transport due to turbulence is able to be examined by considering a flow with spatial gradients of concentration. Experimental literature regarding particle laden jets are provided in references (Ferrand et al., 2003; Prevost et al., 1996), mixing layers (Hishida et al., 1992; Y. Yang and Troutt, 2000), wakes (Bagherpour, 2015; Borée et al., 2001) and boundary layers (Borée et al., 2001). These studies, notably the work of Y. Yang and Troutt (2000), demonstrate that gradients of mean droplet concentration produce “preferential concentration” described to occur incidentally within dissipative eddies.

The main instrument used in this research, the Phase Doppler Interferometer (PDI) was used in the studies of Bateson and Aliseda (2012); Borée et al. (2001); Gerashchenko et al. (2011); Hishida et al. (1992); Prevost et al. (1996); Bagherpour (2015). The PDI allows the measurement of the droplet sizing and 3-components of
Bagherpour performed an experiment described in Bagherpour and Holloway (2015) which consisted of spray released in the wake of a disk. The experiment did not include significant gravitational effects. Bagherpour was able to determine turbulent transport properties of the droplets similar to the objectives of the present study. The wake was axisymmetrical and full 3-D velocity measurement were obtained. The proposed research will perform a similar study for grid turbulence which necessitates only two-velocity components to be measured, due to the isotropy of the turbulence structure within the flow. The end goal is measurement of the inter-arrival times of particles, diameter and velocity and statistics of droplet transport to support formulation of numerical and theoretical methods.

There are few experimental studies of turbulent droplet transport available in the literature that include significant gravitational effects; more specifically, studies of gravitational effects within grid turbulence (which is approximately isotropic and homogeneous) with a spray plume originating from a point source.

The experiment of Bateson and Aliseda (2012) is most relevant to the present experiment in terms of set-up. They were interested in the preferential concentration of droplets in warm-rain clouds. The turbulence conditions were produced using a grid in a wind tunnel. Similar to the proposed experiment, Bateson and Aliseda (2012) used a Phase Doppler Interferometer (PDI). However, the spray used in the experiment by Bateson and Aliseda (2012) was distributed homogenously across the test section. The droplet size range for this experiment was 10-200 $\mu m$ with a volume fraction, $\alpha_v = 2.7 \times 10^{-5}$. The spray was introduced using a complex manifold injection system with injection points at each node on the grid instead of a single point source used in the presented research in this paper. It was determined by Bateson and Aliseda (2012) that small droplet trajectories and coalescence are strongly influenced by background air turbulence. The radial distribution function (RDF) was found
to have a significant role in collision kernel models, namely that there was a large
peak below the Kolmogorov microscale and increased local concentration within the
inertial length scale (~10-100 Kolmogorov lengths).

The equation of motion for a single droplet can be represented as (Bagherpour,
2015):

\[
\frac{d\vec{V}}{dt} = \frac{18\mu_c C_D Re_d}{\rho_d d^2} \frac{24}{24} (\vec{U} - \vec{V}) + \vec{g}
\]  

(1.1)

where

\[
Re_d = \frac{\rho_c d |\vec{U} - \vec{V}|}{\mu_c}
\]  

(1.2)

and \(\vec{U}\) is the carrier phase velocity and \(\vec{V}\) is the dispersed phase velocity. \(C_D\) is the
drag coefficient. Bagherpour (2015) cites a work by Schiller and Naumann to relate
the drag coefficient to the Reynolds number.

\[
C_D = \frac{24}{Re_d} (1 + 0.15 Re_d^{0.687})
\]  

(1.3)

Substituting Eq. 1.3 into Eq. 1.1 yields an equation for the droplet motion with
respect to time. The momentum response time, \(\tau_d\) is the time it takes for the relative
velocity to decrease to the 63\% of its initial value. For higher Reynolds numbers
\((1 < Re_d < 800)\):

\[
\tau_d = \frac{\rho_d d^2}{18\mu_c} (1 + 0.15 Re_d^{0.687})^{-1}
\]  

(1.4)

It is important to note that the Reynolds number will be \(Re < 3\) for a droplet size
range of 10-100 \(\mu m\). Eq. 1.4 is difficult to use since \(Re_d\) cannot be calculated without
simultaneous coincident velocities of the carrier phase and dispersed phase used for
determining the relative velocity. Therefore the limiting Stokes form valid for \(Re < 1\)
will be used to characterize the droplet time scales.

\[
\tau_d = \frac{\rho_d d^2}{18\mu_c}
\]  

(1.5)
The Stokes number is the ratio of droplet response time to time scales of the flow, using either the time scale of the larger energy containing eddies or the smaller scale dissipative eddies. These are defined as

\[ St_e = \frac{\tau_d}{\tau_e} \tag{1.6} \]
\[ St_k = \frac{\tau_d}{\tau_k} \tag{1.7} \]

respectively. The energetic time scale of the turbulence is calculated from:

\[ \tau_e = \frac{L}{u'_z} \tag{1.8} \]

where \( L \) is the grid spacing (0.25” or 6mm) and \( u'_z \) is the rms of the gas phase fluctuating velocity component.

The droplets that move through turbulent gas flows have a particle velocity, \( V_i(t) \) and travel along a trajectory, \( X_i(t) \) with time, \( t \). The variance of droplet position, \( x_j = X_j - \overline{X_j} \), relative to the ensemble average trajectory, \( \overline{X_j}(t) \) has the following functional dependence (Fung et al., 2003):

\[ \overline{x_i(t)x_j(t)} = F_{ij}(St_e, \frac{g\tau_d}{v}, \frac{d}{\eta_k}, Re_l, \frac{tv}{l}) \tag{1.9} \]

where \( St_e = \tau_d/\tau_e \) is the Stokes number based on the energy containing turbulence time scale. The concentration-velocity correlation is a basis for droplet transport and is a function of the aforementioned dimensionless parameters:

\[ \overline{c_i v_i} = F_i(St, \frac{g\tau_d}{v}, \frac{d}{\eta_k}, Re_l) \tag{1.10} \]

Regarding particles on the order of the Kolmogorov length scale or larger (as in the present study), an alternative turbulence time scale, \( (d^2/\epsilon)^{1/4} \), was recommended
(Monchaux et al., 2012; Xu and Bodenschatz, 2008). Similarity of the Stokes number seen in Eq. 1.7 means that a small-scale wind tunnel experiment and the full scale model will have similar droplet response times in relation to their respective time scales. Several dimensionless parameters must be equated for the two flows: \( St_k, \frac{\varrho a}{\nu}, \frac{d}{\eta_k} \) and \( Re_l \).

Analytical studies of Eq. 1.9 in homogeneous turbulence (Batchelor, 1949; Durbin, 1980; Fung et al., 2003; Reeks, 1991, 2005; Swailes and Darbyshire, 1999) are based on the assumption of a Gaussian distributed particle velocity along its trajectory. Where time, \( t \) is large and the effects of gravity are neglected, the variance of either a fluid element or particles can be determined by (Monin and Yaglom, 1971):

\[
\overline{x_i(t)x_j(t)} = 2 \sqrt{v_i^2 v_j^2 \gamma_{ij} t} \tag{1.11}
\]

where \( v_i = V_i - \nabla_i \) is the particle velocity and \( \gamma_{ij} \) are the Lagrangian integral time scales of \( \overline{v_i v_j(\tau)} \). It has been proposed that a Gaussian distributed particle velocity field has broader generality than a single phase velocity field (Moreau et al., 2009).

Equations 1.9 and 1.11 have been thoroughly investigated with Direct Numerical Simulation using fully resolved homogeneous turbulent flow fields in conjunction with ensemble averaged Lagrangian point-particle tracking methods. Numerical simulations (Bragg and Collins, 2014; Gualtieri et al., 2013; Pascal and Oesterlé, 2000; Saw et al., 2012; Sundaram and Collins, 1999; Wang and Maxey, 1993) include examples of isotropic and sheared turbulence with and without gravitational effects for initially homogeneous particle distributions. These fundamental studies have focused on dilute mixtures of particles having mean volume concentrations, \( \overline{C_v} < 10^{-3} \), and flow conditions giving, \( \frac{d}{\eta_k} << 1; \frac{\varrho a}{\nu} < 3, St < 6 \) and \( Re_\lambda < 200 \). A significant finding of the work was "preferential concentration" of droplets at scales comparable with the dissipation scales of the turbulence (Kolmogorov); an effect that is maximum at \( St \sim \)
1, but diminishes with increasing $\rho_d \overline{C_v}/\rho$ (Gualtieri et al., 2013). Particles with St $\sim 80$ were also observed to concentrate at scales on the order of the turbulence integral scale (et al., 2014).

Regarding the effects of gravity, Maxey (1987) notes that there have been numerical simulations and experiments done by Wells & Stock (1983), Riley (1971), Riley and Patterson (1974) and Reeks (1980) showing that there was no change in settling velocities due to the addition of turbulence. However, he notes that these experiments and simulations were primarily for particles with a significantly smaller turbulent rms velocity when compared with the settling velocity or with mean zero settling velocities. Numerically, he determined particles in a random cellular flow field were found to settle out more rapidly than in still fluid owing to the inertial effects. With minimal inertia the particles settle on average at the same rate as in still fluid.

### 1.3 Statistical Properties of Droplet Sprays

Concentration is an important feature of any polydisperse droplet spray. The spatial number concentration, $C$, (or number density) is defined as

$$C = \lim_{\delta V \rightarrow \delta V^o} \frac{\delta N}{\delta V}$$

(1.12)

where $\delta N$ is the number of elements of the material in volume, $\delta V$ and $\delta V^o$ is the limiting volume for which the continuum assumption is valid (Crowe et al., 2011). The volume concentration is similar to the spatial number concentration, but instead counts the volume of liquid material, $\delta V_l$ in observation volume $\delta V$.

$$C_{v,i} = \lim_{\delta V \rightarrow \delta V^o} \frac{\delta V_l}{\delta V}$$

(1.13)

For a dilute spray the limiting volume, $\delta V^o$ is much larger than in the cases of
single phase flow. Thus, the carrier phase is continuous on much smaller scales than
the dispersed phase. The distribution of droplets in space can be approximated as
a Poisson distribution for which the variation in the number of droplets in a given
volume can be calculated as (Landau and Lifshitz, 1996)

$$\frac{\sigma(\delta N)}{E(\delta N)} \approx \frac{1}{\sqrt{E(\delta N)}}$$

where \(\sigma\) is the standard deviation. If for example a variation of 1 % is desired then it
follows that the expected number of droplets must be approximately \(10^4\). Bayesian
methods have also be applied, because in the case of a dilute spray this limiting
volume (to produce a 1% variation) is the same scale as the flow domain.

The flux, both number flux and volume flux, is a property of the flow which
describes the transport rate per unit area. Droplet number flux is defined for a
limiting area and time interval as:

$$\vec{\Phi}_N(t) = \lim_{dA,dt \to 0} \frac{E\left(\sum_{i=1}^{N_d} \vec{e}_{V,i} \cdot \vec{e}_{dA}\right)}{|dA|dt}$$

Flux is related to the concentration using the droplet velocity distributions. The
average number flux of droplets at position, \(x\), in a time interval, \(\Delta t\), can be related
to the average number of droplets in a time interval as

$$\overline{\Phi}_N(t, x) = \frac{\int_t^{t+\Delta t} \vec{\Phi}_N(t, x) \cdot \vec{dA} dt}{|dA|\Delta t} = \frac{E\left(\sum_{i=1}^{N_d} \vec{v}_{V,i} \cdot \vec{e}_{dA}\right)}{|dA|dt}$$

Averaging of droplet velocity properties can be done using two different methods:
Reynolds averaging and Favre averaging. Reynolds averaging for a continuous velocity
Reynolds decomposition of $V$ is the summation of the average velocity, $\overline{V}$ and the fluctuating component, $v$.

$$V = \overline{V} + v$$  \hspace{1cm} (1.18)$$

where $\overline{v} = 0$. This can also be performed for any continuous random variable that varies in time.

Favre averaging of sprays takes the form of the volume concentration weighted time average (Favre, 1983)

$$\tilde{V} = \frac{1}{T\overline{C}_v} \int_0^T C_v(t)\overline{V}(t)\,dt$$  \hspace{1cm} (1.19)$$

Similar to Reynolds decomposition, the instantaneous quantity can be expressed as

$$V = \tilde{V} + v_F$$  \hspace{1cm} (1.20)$$

with $v_F$ as the fluctuating component. Note that the Favre average of the fluctuating component, $\overline{v_F}$, is not necessarily zero.

The relation between Reynolds averaged and Favre averaged velocity is as follows.
\[
\bar{V} = \frac{1}{TC_v} \int_0^T C_v(t) V(t) dt
\]
\[
= \frac{1}{TC_v} \int_0^T (\bar{C}_v + c_v(t)) (\bar{V} + v(t)) dt
\]
\[
= \frac{1}{TC_v} \left( \bar{C}_v \bar{V} T + \int_0^T (c_v(t))(v(t)) dt \right)
\]
\[
= \bar{V} + \frac{\bar{c}_v v}{C_v}
\]

which yields the equation for the turbulent transport of droplet concentration

\[
\bar{c}_v v = C_v (\bar{V} - \bar{V})
\]

In this study it will be assumed that the droplets are small enough to remain spherical, that the ratio of the density of the dispersed phase to the density of the carrier phase is much greater than one and that evaporation and collision can be neglected. This means that the drag between the two phases and the gravitational forces will be the dominating factors in the droplet motion.

### 1.4 Research Objectives

The objective of the present research was to measure the droplet dispersion from a point source in isotropic grid turbulence in the presence of gravity. It is a model of the inner scales of atmospheric turbulence that would be modelled as subgrid in an LES simulation of the atmosphere. Experimental conditions were chosen to match the conditions that correspond to a 20 km/hr wind at a 30 m elevation (Lumley and Panofsky, 1964).
Table 1.1 shows a comparison between the experiment scales of interest in the UNB wind tunnel and the actual lower atmospheric boundary layer which would be modelled in an LES model. $l$ represents the characteristic length scale, $\Delta$ is the LES grid resolution, $v_\Delta$ is the turbulence on the LES grid scale, $\eta_k$ and $\tau_k$ are the Kolmogorov length scale and time scale, respectively.

Table 1.1: Model scales for grid turbulence versus atmospheric scales

<table>
<thead>
<tr>
<th></th>
<th>$l$ (m)</th>
<th>$v$ ($\frac{m}{s}$)</th>
<th>$Re_l$</th>
<th>$\tau_k$ (ms)</th>
<th>$\eta_k$ (mm)</th>
<th>$\Delta$ (m)</th>
<th>$\frac{v_\Delta}{\eta_k}$</th>
<th>$Re_\Delta$</th>
<th>$d$ (µm)</th>
<th>$\tau_d$ (ms)</th>
<th>$St_k$</th>
<th>$St_e$</th>
<th>$\frac{\tau_d}{v}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmos. BL</td>
<td>12</td>
<td>1</td>
<td>7.6 x $10^5$</td>
<td>13</td>
<td>0.46</td>
<td>1</td>
<td>0.44</td>
<td>2.8 x $10^4 - 15$</td>
<td>0.049</td>
<td>0.0003</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grid Turb.</td>
<td>0.006</td>
<td>0.20</td>
<td>77</td>
<td>3.4</td>
<td>0.23</td>
<td>-</td>
<td>-</td>
<td>10 - 140</td>
<td>0.30</td>
<td>- 0.10</td>
<td>- 2.85</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tab. 1.1 shows good agreement between the Kolmogorov Stokes number, $St_k$ for both the atmospheric boundary layer and the grid turbulence. Neither of which can be measured experimentally. The energy containing Stokes number, $St_e$ shows some slight overlap; the largest Stokes number for droplets in the atmospheric boundary layer is equivalent to the smallest Stokes number in the grid turbulence experiment. It is more important that the small-scale Stokes numbers correspond, considering that the large-scale Stokes numbers can be determined experimentally. Point process droplet dispersal statistical information such as mean size, velocity, cross-correlations of velocity signals from both measured channels and correlations of concentration and velocity were collected. The influence of gravity was observed through size dependent concentration and flux profiles, droplet velocity distributions and velocity-concentration correlations along the length of the test section.
Chapter 2

Experimental Apparatus

2.1 Wind Tunnel Configuration

The experiments for the present study were conducted in the turbulence laboratory in Head Hall at the University of New Brunswick. A 20 horse-power motor with a variable speed drive controls a fan able to produce velocities up to $30 \, \text{m/s}$. The test section is a square channel with a 58 cm x 58 cm cross-section and 3 m in
length. The walls are transparent polycarbonate such that optical access is allowed. A 10 Torr pressure transducer was used to monitor the pressure drop across the tunnel contraction and hence monitor wind tunnel speed. The mean tunnel velocity is calculated from the following equation

\[ V(m/s) = 14.86\sqrt{E - E_o} \]  

(2.1)

where \( E - E_o \) is the voltage reading from the transducer in Volts.

The spray nozzle system comprises the nozzle itself, a streamlined supporting bracket to minimize vortex shedding and turbulent boundary layer growth within the wind tunnel. The spray fluid is provided by a pressure vessel with regulators, valves and connecting lines. Fig. 2.2 shows the spray assembly with the exception of connecting lines, regulators and pressure vessel from the inside of the wind tunnel. The view in Fig. 2.2 is a section of the settling chamber of the tunnel seen in Fig. 2.1. The spray assembly is attached to the wall of the wind tunnel contraction and the nozzle is placed approximately three feet from the start of the test section. The premise of the design is to minimize the wake created by the support assembly by forcing it through the contraction.

The nozzle used was the BETE PJ10 Inpingement nozzle seen in Fig. 2.3. It was chosen because it produces a high percentage of droplets under 50 \( \mu \)m which met the objective droplet ranges, 12 - 96 \( \mu \)m. The specifications for the nozzle are shown in Fig. A.5. The spray liquid was supplied by a pressure vessel at 60 psi filled with water and soap. The pressurized container was chosen as it is designed to produce a steady flow for approximately 30 minutes at a flow rate of 0.021 gallons per minute. A 1/2” x 1’ rectangular aluminium tube with 1/16” wall thickness is fixed to wooden cap on the outside of the wind tunnel as seen in Fig. 2.2. The purpose of the rectangular bar is to produce torsional rigidity as well as allowing the tube carrying the spray liquid
a path to the nozzle at the end of the circular aluminium tube (ID 0.370" OD 0.5"). The nozzle was secured with 1mm fishing line to prevent unwanted vibration. Three fishing lines were attached at 120° to the nozzle to ensure stability. The support arm had a cross-section of a NACA 0020 airfoil made of 3D printed plastic slid over the rectangular bar, as seen in Fig. 2.4. The required diameter of the lines to the spray nozzle were determined from pipe friction as described in Fig. A.5. The required diameter of the supply line was 3/32”.

The grid used to generate turbulence was a 304 stainless steel woven wire cloth. The wire cloth had an open area of 62%, a wire diameter of 1.4 mm and a mesh spacing of 5 mm. The stainless steel mesh was sprayed with a water-repellant silicone
to prevent surface tension from attaching liquid water to the grid. To further inhibit droplet attachment detergent was added to the spray liquid. This combination resulted in a significant reduction in the nozzle wake size.

The wind-tunnel speed was chosen such that there was sufficient convection time
in the windtunnel for the heavy droplets to sink. The sink velocity is

\[ V_{\text{sink}} = \frac{\rho_d d^2 g}{18 \mu_c} \left[ 1 + 0.15 \frac{Re_d^{0.687}}{Re_d} \right]^{-1} \quad (2.2) \]

where \( Re_d \) is the Reynolds number based on the sink velocity, \( V_{\text{sink}} \) and droplet diameter, \( d \). This suggests that the small droplets will have a smaller sink rate number than the larger droplets. The goal is to demonstrate the differential sinking behaviour of a broad spectrum of droplet sizes. Iterative calculations were performed using the sink rate and test section length until a desirable droplet trajectory was achieved, showing separation of small droplets from the larger droplets. A windtunnel air speed of 10 m/s was determined to be optimal for droplet separation, but this speed allowed some droplets to coalescence on the stainless steel grid. The attachment phenomenon was found to be greatly reduced above 13.5 m/s so this was the selected testing velocity.

### 2.2 Phase Doppler Interferometry

This section describes the Phase Doppler Interferometry and carriage system used to facilitate droplet velocity and size measurements (See Appendix A). Considerations include the reference axes, the lab-space, the wind tunnel test cross-section as well as the spray nozzle system.

The apparatus for measurement can be seen in Figure 2.5. This orientation allows movement along all three desired axes. Note that the carriage is able to slide along the length of the wind tunnel and the transmitter and receiver are attached via a 40-gage chain and sprockets and able to slide along the vertical transverse plane.

The transmitter and receiver both are equipped with the 50 cm focal length lens provided by Artium Technologies Inc. The collection angle was set to 30° as recommended in the PDI manual (Artium Technologies Inc., 2009). The receiver is moved
horizontally by rotating the adjustment dials on the side of the transmitter, but is restricted to a 10 cm range. Additional horizontal points can be measured by rotating the receiver. The receiver needs to be rotated because of the fixed focal length of 50 cm. More information on the set-up of the receiver and transmitter can be found in the results section of this document and Appendix.

Data collection and processing were done using the Artium Technologies proprietary software package, AIMS. The signal conditioners shown in Fig. 2.6 process the raw signals to produce the size and 2 components of droplet velocity. They also control the aperture on the receiver to maintain optimum size resolution.

2.3 Data Processing

All data processing for the results contained within this document was performed with the aid of MATLAB2013a. Gathered data was available for export from AIMS to data files that were processed in Matlab. Time information, coincidental velocity measurements, droplet diameter, Sauter mean diameter, D30 and mean volume flux were all exported from AIMS as .m files for easy importing into MATLAB. As men-
tioned within the research objectives, it is of interest to find the correlations between volume concentration, $C_v$ and droplet velocities $V_i$. Velocity correlations $\overline{v_i v_j}$ can be determined from 2 coincident velocity components with the current set-up, vertical (y-direction) and streamwise (z-direction). Vertical velocity is of particular interest because of gravitational effects. The volume concentration can be calculated from the flux as follows:

$$C_v = \Phi_v \bar{V}$$  \hspace{1cm} (2.3)

The Reynolds and Favre average velocity are calculated from the PDI data accounting for the length bias present insofar that slower droplets are sampled less than faster droplets. To account for this effect the average is denoted as $\bar{V}$ and is weighted by the droplet interarrival time. Bagherpour (2015) shows how the accuracy of this method of weighting by the interarrival time is comparable to other contemporary methods.
It can be calculated as follows:

\[
\bar{V} = \frac{\sum_{i=1}^{N_{PDI}} \tau_i V_i}{T_i}
\]  

(2.4)

The Reynolds rms value can be calculated from

\[
v' = \left[ \frac{\sum_{i=1}^{N} \tau_i (V_i - \bar{V})^2}{T} \right]^{1/2}
\]

(2.5)

where \(\sum_{i=1}^{N}\) represents a summation across all points in the sample population. The Favre average is calculated as

\[
\tilde{V} = \frac{\sum_{i=1}^{N_{PDI}} d_i^3 V_i}{\sum_{i=1}^{N_{PDI}} d_i^3 |V_i|}
\]  

(2.6)

and the concentration is determined from the flux from

\[
\overline{C_v} = \Phi_v \frac{\sum_{i=1}^{N_{PDI}} d_i^3 V_i}{\sum_{i=1}^{N_{PDI}} d_i^3 |V_i|}
\]

(2.7)

The probability density function (pdf) of droplet diameters throughout the flow, \(f_d\) was calculated from:

\[
f_d(i) = \frac{N_i}{\sum_{i=1}^{k} N_i \Delta d}
\]

(2.8)

where \(N_i\) is the number of droplets in size bin, \(k\) is the total number of bins and \(\Delta d\) is the binwidth. The probability distribution function, \(f_v\) for any velocity velocity, is determined from the follow equation for each bin, \(i\):

\[
f_v(i) = \frac{N_i}{\sum_{k=1}^{N_i} \tau_k \Delta V_z}
\]

(2.9)

where \(N_i\) is the total amount of droplets in each bin, \(i\), \(\tau_k\) is the interarrival time for each measurement, \(k\). \(T\) is the total elapsed time for the sample population of
interest and $\Delta V_z$ is the binwidth.

The streamwise component of the dispersed phase velocity is used in conjunction with the probe area (PVC), receiver focal lengths, aperture slit width and collection angle to determine the volume flux for a measurement set. AIMS is able to export the mean volume flux in the streamwise direction as well as the PVC (probe volume correction), which is determined from the ratio of the maximum size class to current measurable size class. The mean volume flux, for all size classes, leads to the mean volume flux for the size class of interest with the aid of the PVC through the following relationship:

$$\Phi_d = \Phi_{tot} \frac{P}{PVC_{avgtot}}$$  \hspace{1cm} (2.10)

where $P_d$ is the ratio of droplets in size class, $d$ to the total observed droplets in a measurement set. The concentration can be calculated from Eq. 2.11 (for the size class of interest) and Eq. 2.10 using the average probe volume correction of the size class.

$$C_d = \frac{\Phi_d}{\tilde{V_d}}$$  \hspace{1cm} (2.11)

where $\tilde{V_d}$ is the Favre average velocity, or concentration-weighted velocity.
Chapter 3

Results

3.1 Test Conditions and Overview

All measurements were conducted at a steady wind tunnel speed of 13.5 m/s. Measurements were made on a grid of 5x15 points in the xy-plane at four different downstream stations ($z = 362$ mm, $1143$ mm, $1403$ mm and $2083$ mm). Fig. 3.1 is drawn to scale to show the fraction of the cross-section covered by the measurements. The points were spaced at 12.7 mm (0.5") with exceptions in the two lowest measurable positions where 25.4 mm increments were used. The 4 different downstream measurement cross-sections were denoted as stations 1-4. These four positions were chosen because of the development of the flow and they were the locations which presented minimal optical obstructions. In order to measure the full extent shown of the vertical range shown in Fig. 3.1 two different receiver set-ups were necessary. To measure the y-positions 50.8 mm and above the receiver was shifted to the third quadrant and to measure all other positions the receiver was shifted to the second quadrant. Note that the 50.8 mm position is the geometric centerline of the test section whereas the centerpoint of the coordinate system is matched with the center of the spray plume. The corresponding PDI angles are found in Table 3.1. The angles and quadrants
referenced are based on the coordinate system defined in Fig. 3.2 and 3.1.

Figure 3.1: Test section cross-section at detailing the measurement positions. Origin of the coordinates placed at the nozzle location. Dimensions in millimetres.

In order to avoid complex transformation matrices the two droplet velocities measured in the receiver, $V_1$ and $V_2$ were aligned with the streamwise and vertical velocity of the droplets, respectively. Each data set for a single position consisted of 2000 datapoints to provide sufficient statistical information for bin widths of 3 $\mu m$ (ie >15 drops in each bin). The design of the PDI carriage allowed for vertical and streamwise movement, keeping the transmitter and receiver mostly aligned with each other.
each downstream station measurements were obtained at every y-position, then repeated at the next downstream location following a confirmation of alignment via the aperture of the receiver and an oscilloscope viewing of the signal from the photodetectors in the receiver. After all z-stations were recorded the focusing dials on both the receiver and transmitter were adjusted to migrate the collection volume away from the center. This corresponds to (-x) movement; (+z) is in the direction of wind flow and (+y) is upwards. The coordinate system used by the receiver is different from that used for reporting the measurements. (See Section 3.2).

Table 3.1: PDI Transmitter and receiver angles for various x-positions

<table>
<thead>
<tr>
<th>x</th>
<th>$y &lt; 50.8$ mm</th>
<th>$y \geq 50.8$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_T(\circ)$</td>
<td>$\theta_R(\circ)$</td>
</tr>
<tr>
<td>+1”</td>
<td>0</td>
<td>152</td>
</tr>
<tr>
<td>0” (cl)</td>
<td>0</td>
<td>152</td>
</tr>
<tr>
<td>-1”</td>
<td>0</td>
<td>152</td>
</tr>
<tr>
<td>-2”</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>-3”</td>
<td>0</td>
<td>150</td>
</tr>
</tbody>
</table>

In addition to the PDI measurements, experiments with a hotwire were performed separately in the wind tunnel without the spray to study the nozzle wake and grid turbulence. The wake was found to be very small (see Section 3.2 for Gas-phase Results). The small wake results from the acceleration through the wind tunnel contraction and pressure drop across the grid. The PDI measurements confirmed that the droplet range observed in the experiment coincides with the range which is desired. High beam coincidence was achieved for all measurement locations, so covariance and correlations were able to be calculated for streamwise and vertical velocities.
Figure 3.2: Schematic showing the PDI arrangement of transmitter and receiver angles with the transmitter aligned axes.
### 3.2 Gas-phase Velocities

The gas-phase velocity and turbulence due to the presence of the grid and the spray apparatus were measured with a hotwire, horizontally and vertically in increments of 1 cm, through the origin of the coordinate system defined in Fig. 3.1. The procedure was repeated for 3 PDI measurement locations, z/L = 57, z/L = 220 and z/L = 328. The measurements for the case without the grid (red circles) were only performed at z/L = 57 and were measured in increments of 5 cm. Fig. 3.3 shows that even without the grid the wake of the nozzle and support structure is minimal and closely matches the corresponding velocity with the grid in place. The wake in the mean velocity with the grid in place is also very small. There is, however, an anomaly at z/L = 328, near the end of the wind tunnel.

![Figure 3.3: Mean gas-phase velocity through horizontal centerline. (a.) z/L = 57 b.) z/L = 221 c.) z/L = 328)](image)

Fig. 3.4 provides the data from Fig. 3.3 along the vertical line passing through.
the centerline for three of the four streamwise locations. There does appear to be a discrepancy between the results with and without the grid in place near the upper and lower walls of the measurement plane.

The turbulence intensity of the gas-phase velocity is shown in Figure 3.5 and 3.6. At z/L = 57 the turbulence is approximately uniform at 1.8% with the grid in place and the turbulence is around 0.1% without the grid in place. The turbulence intensity increases to around 0.25% near the centreline perhaps due to a slight wake from the nozzle. Overall the turbulence decays as the flow travels downstream. Fig. 3.6 shows the behaviour of the gas-phase along vertical lines through the centerline. Similar behaviour can be seen to Fig. 3.5 with and without the grid. This vertical centerline profile view shows how much stronger the gas-phase velocity fluctuations are with the grid in place. In Fig. 3.6 is it obvious that the uniformity seen in Fig. 3.5 along the
Figure 3.5: Turbulence intensity of gas-phase velocity across horizontal centerline. (a.) $z/L = 57$ (b.) $z/L = 221$ (c.) $z/L = 328$)

vertical direction is not as good as in the horizontal direction.
Figure 3.6: Turbulence intensity of gas-phase velocity across vertical centerline. (a.) $z/L = 57$ b.) $z/L = 220$ c.) $z/L = 328$)
3.3 Droplet Size

The hotwire results confirmed that the gas-phase flow was nearly homogeneous with weak mean flow gradients only present at the edge of the measured region. The two-phase flow was measured with the PDI instrument shown in Fig. 2.5.

Centerline droplet size distributions can be seen in Fig. 3.7 in terms of the percentage of volume and probability. As the results show, the probability distribution is skewed towards the smaller droplets; with the majority in the 25-35 micron range. In terms of volume, however, the 80-110 $\mu m$ droplets make up around 50% of the total measured volume. Although the probability distribution shows that the larger droplets occur more infrequently than the smaller droplets on the centerline, close to the grid, there is a significantly greater contribution to the total volume. The spray plume therefore consists of droplets within the desired range of 10-140 $\mu m$ as stated in the research objectives. The spray as it travels downstream will be described in the subsequent sections in terms of probability distributions of different size classes for different downstream stations. Mean streamwise and vertical velocity, correlation and turbulence intensity plots will also be presented.

Vertical profiles of the mean volume diameter at the designated downstream positions are provided in Figs. (3.8 - 3.11). Fig. 3.8 shows the mean volume diameter at the station closest to the grid for all y (vertical) and x (horizontal) positions, normalized by grid spacing, L. The nozzle is aligned with (x,y) = (0,0). Fig. 3.8 shows that near the grid there is a concentration of small droplets near the centerline with large droplets around the edges. As the flow moves further downstream the spray plume expands and the small droplets become more uniformly distributed. One can also see at the furthest downstream position that the largest droplets have migrated downwards.

In observing Fig. 3.11, one may say there is not a large spread in overall averages, and the variance, appears to increase towards the center of the flow at $x/L = 0$. 29
Figure 3.7: Centerline probability distribution of diameter and percentage of total volume at $z/L = 57$. (a) $y/L = 2$ b) $y/L = 0$ c) $y/L = -2$

There is an almost parabolic shape in $d_{30}$ at this position. Figures 3.8 - 3.10 follow similar behaviour, but it becomes obvious at the furthest downstream position, Fig. 3.11, that the larger droplets, $d > 110$ microns are near the bottom of the test section and drop out of the measurement zone with increased downstream distance. As a consequence of this migration, closer to the grid the droplet size profile is more symmetric along a vertical line than they appear downstream.

Figures 3.12-3.15 are the probability density functions of droplet size located along the vertical centerline at different downstream positions from the grid. Note that the plots include droplets of all velocities. They clearly show how the large droplets are separated by gravity as they fall downwards with downstream distance.

At $z/L = 57$, the plume appears to be relatively symmetric with larger droplets equally distributed at the upper and lower edges. The larger droplets on the edge of the plume may be due to the breakup of the liquid sheet formed by the nozzle anvil.

As the spray plume travels downstream it can be seen in Figs. 3.13-3.15 that
the larger droplets are depleted near the top of the flow and more large droplets are found near the bottom of the measurement plane. Fig. 3.13a shows that there is still a large spread in droplet diameters in the following downstream position. However, in the next downstream position a decrease of larger droplets in Fig. 3.14a, b and c is observed. The large droplets that were visible upstream in Figure 3.14g have moved.

Figure 3.8: Mean volume diameter for droplets of all velocities at various cross-stream locations for $z/L = 57$.

Figure 3.9: Mean volume diameter for droplets of all velocities at various cross-stream locations for $z/L = 180$. 
down to Figure 3.15h, from $y/L = -10$ to $y/L = -18$.

Fig. 3.15 shows the centerline $y$-$z$ plane droplet size probability distribution at the furthest downstream position. At the top positions there were very few droplets available to be recorded. The scattered light intensity from these droplets was low and this, in turn, made it difficult to line up the aperture of the receiver with the
beam focal point. In some cases this created an issue in matching the measurement field illustrated in Fig. 3.1. For example, at $z/L = 57$, the two bottom measurement positions could not be recorded, due to the aforementioned reason of insufficient data.

In order to study the movement and velocity statistics of droplets of different sizes the entire droplet size range was divided into 4 classes. These were chosen somewhat arbitrarily to identify the size classes that show significant sinking effects (large) and those that experience significant turbulent diffusion.

Table 3.2: Size categorization of droplet diameter.

<table>
<thead>
<tr>
<th>Size Class</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>Extra Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Droplet Range } \mu m$</td>
<td>$d &lt; 25$</td>
<td>$25 \leq d &lt; 40$</td>
<td>$40 \leq d &lt; 60$</td>
<td>$d \geq 60$</td>
</tr>
<tr>
<td>$\overline{St_d} = \frac{\tau_d}{\bar{\nu}}$</td>
<td>0.0890</td>
<td>0.1881</td>
<td>0.4718</td>
<td>1.7034</td>
</tr>
<tr>
<td>Number Fraction (%)</td>
<td>11.8</td>
<td>25.9</td>
<td>24.7</td>
<td>37.6</td>
</tr>
</tbody>
</table>

The following sections show results which have been filtered by size class and Tab. 3.2 shows the representative droplet diameter ranges for the different size classes.
Figure 3.12: Centerline probability distribution of droplet size for all droplet size classes for z/L = 57. (a.) y/L = 14, (b.) y/L = 10, (c.) y/L = 6, (d.) y/L = 2, (e.) y/L = 0, (f.) y/L = -2, (g.) y/L = -6, (h.) y/L = -10)
Figure 3.13: Centerline probability distribution of droplet size for all droplet size classes for z/L = 180. (a.) y/L = 14, (b.) y/L = 10, (c.) y/L = 6, (d.) y/L = 2, (e.) y/L = 0, (f.) y/L = -4, (g.) y/L = -10, (h.) y/L = -18)
Figure 3.14: Centerline probability distribution of droplet size for all droplet size classes for z/L = 221. (a.) y/L = 14, b.) y/L = 10, c.) y/L = 6, d.) y/L = 2, e.) y/L = 0, f.) y/L = -4, g.) y/L = -10, h.) y/L = -18)
Figure 3.15: Centerline probability distribution of droplet size for all droplet size classes for top half of $z/L = 328$. (a.) $y/L = 12$, b.) $y/L = 8$, c.) $y/L = 4$, d.) $y/L = 0$, e.) $y/L = -2$, f.) $y/L = -6$, g.) $y/L = -10$, h.) $y/L = -18$)
3.4 Droplet Velocity

Fig. 3.16 shows the probability density function of the droplet streamwise for all positions measured within the spray plume. This figure is not sorted by size class so that it represents the entire velocity population. The Gaussian distribution (dashed line) was fitted to the population data using the Reynolds averaged mean, $\overline{V}$ and rms of fluctuating component, $v'$ which are calculated from Eq. 2.4 and 2.5, respectively.

The Reynolds averaged mean of the streamwise droplet velocity, $\overline{V}_z$ is 13.5 m/s which matches the mean gas-phase velocity within the wind tunnel. The data does not match the fitted Gaussian insofar that the data is skewed to the right of the mean. It is also substantially flatter than the normal curve. The probability density distribution for all of the vertical droplet velocity data (Fig. 3.17) appears less skewed and flatter than the streamwise velocity curve.

In Figure 3.18 the velocity distribution has been separated into the size classes shown in Tab. 3.2: small, medium, large and extra large. From this figure it is
Figure 3.17: Probability distribution of vertical droplet velocity for all droplet size classes at all positions.

It is evident that as the Stokes number increases the variance becomes slightly smaller. This agrees with the expectation that the Stokes number affects how well droplets respond to fluctuations in the carrier phase velocity. Comparing this figure with the figure containing all of the size classes, Fig. 3.16, it can be seen that the shapes are similar, although as the size class increases the skewness to the right also increases.

Figure 3.19 for the vertical velocity has the same lack of flatness and skewness seen previously, but the implementation of size filters allows the observation of the strong downward velocity seen in the extra large size class, \( d > 60 \mu m \), Fig. 3.19d. The other size classes show a small downwards velocity, but the largest size class shows that the effects are almost doubled.

Figure 3.20 contains the same size classes as 3.18, but only the data on the x-y plane at \( z/L = 57 \), which is the position closest to the nozzle. Similar to the Fig. 3.18, the smaller droplet sizes classes, i.e. Figs. 3.20a-c, show skewness.

Considering Figs. 3.24 and 3.26 at \( z/L = 221 \) and \( z/L = 328 \), the droplets appear
Figure 3.18: Probability distribution of streamwise droplet velocity for various droplet size classes at all positions. a.) $d < 25$ b.) $25 \leq d < 40$ c.) $40 \leq d < 60$ d.) $d \geq 60$

to be more normal and less skewed towards lower speeds than the other $z$ locations.

The turbulence is also responsible for the spread in the droplet diameters seen in Figs. 3.12, 3.13, 3.14, 3.15 and the larger velocities seen in Figs. 3.20 and 3.22. The larger droplets are expected to be less responsive to turbulent structures within the flow because of their larger Stokes number. Figures 3.21, 3.23, 3.25 and 3.27 show the vertical velocity distribution for the droplet size classes shown in Tab. 3.2 at streamwise locations $z/L = 57$, $z/L = 180$, $z/L = 221$ and $z/L = 328$, respectively. The effects of gravity on the vertical velocity are clearly seen in these Figures. Similar to the streamwise velocity distribution Figures 3.20, 3.22, 3.24 and 3.26, the variance decreases with downstream distance, which indicates that the rms value of the turbulence is decreasing. The largest droplets do not see the same magnitude of rms reduction at the downstream positions, noting Figure 3.27d. Their turbulent energy
Figure 3.19: Probability distribution of vertical droplet velocity for various droplet size classes at all positions. a.) $d < 25$ b.) $25 \leq d < 40$ c.) $40 \leq d < 60$ d.) $d \geq 60$

does not dissipate as quickly as the smaller droplets.

The probability density of the velocity in the vertical direction does not large values of skewness and is more normal than the streamwise velocity distribution. An explanation is that unlike the streamwise velocity, the droplets do not need to be accelerated in the vertical direction to match the velocity of the mean flow. The droplets exit the impingement type nozzle with a minute amount of streamwise momentum and experience a force that accelerates the spray plume to match the mean flow. The mean flow does not have a y-component to the velocity, therefore there is no interfacial drag between the two phases in the y-direction.

The properties of the turbulence at downstream locations for both the streamwise and vertical components as well as the corresponding Stokes numbers can be found in Tabs. 3.3 and 3.4. Table 3.3 summarizes the rms velocity in the streamwise
Figure 3.20: Probability distribution of streamwise droplet velocity for various droplet size classes at $z/L = 57$. a.) All size classes b.) $d < 25$ c.) $25 \leq d < 40$ d.) $40 \leq d < 60$ e.) $d \geq 60$

Table 3.3: Root mean squared fluctuating streamwise velocity (Numerator) and mean Stokes number (Denominator) organized by size and streamwise location.
Figure 3.21: Probability distribution of vertical droplet velocity for various droplet size classes at $z/L = 57$. a.) All size classes b.) $d < 25$ c.) $25 \leq d < 40$ d.) $40 \leq d < 60$ e.) $d \geq 60$

Table 3.4: Root mean squared fluctuating vertical velocity (Numerator) and mean Stokes number (Denominator) organized by size and streamwise location.

<table>
<thead>
<tr>
<th>Size Classes</th>
<th>All</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>Extra Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pos. $d &gt; 0$</td>
<td>$d &lt; 25$</td>
<td>$25 \leq d &lt; 40$</td>
<td>$40 \leq d &lt; 60$</td>
<td>$d \geq 60$</td>
<td></td>
</tr>
<tr>
<td>$z/L =$</td>
<td>$u'_y/St_e$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.2211</td>
<td>0.2086</td>
<td>0.1768</td>
<td>0.1819</td>
<td>0.2105</td>
</tr>
<tr>
<td>0.3575</td>
<td>0.0394</td>
<td>0.1062</td>
<td>0.2371</td>
<td>0.9067</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>0.2446</td>
<td>0.2497</td>
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<td>0.2303</td>
<td>0.2198</td>
</tr>
<tr>
<td>0.2187</td>
<td>0.0340</td>
<td>0.0798</td>
<td>0.1781</td>
<td>0.6851</td>
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</tr>
<tr>
<td>180</td>
<td>0.2209</td>
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<td>0.1869</td>
<td>0.1890</td>
<td>0.2053</td>
</tr>
<tr>
<td>0.2798</td>
<td>0.0391</td>
<td>0.0945</td>
<td>0.2072</td>
<td>0.8051</td>
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<tr>
<td>221</td>
<td>0.2032</td>
<td>0.1850</td>
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<td>0.1610</td>
<td>0.1786</td>
</tr>
<tr>
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<td>0.0389</td>
<td>0.1135</td>
<td>0.2561</td>
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</tr>
<tr>
<td>328</td>
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<td>0.1922</td>
</tr>
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<td>0.0382</td>
<td>0.1277</td>
<td>0.2866</td>
<td>1.0723</td>
<td></td>
</tr>
</tbody>
</table>
direction. The rms of the streamwise velocity decreases as the spray plume travels downstream. It can also be seen that the size class makes a slight difference, with the larger size classes generally having smaller standard deviations than the smallest size class, with some statistical variability due to the smaller size class only containing 12% of the total whereas the extra large size class contains around 38%. The Stokes numbers are calculated from Eq. 1.7 and 1.8, using the rms value of the smallest size class to approximate the rms value of the gas-phase. Table 3.4 summarizes the rms velocity in the vertical direction. The same behaviour as the streamwise rms velocity can be seen. Closer to the rms values are approximately half of what they were for the streamwise direction. However, as the flow progresses downstream the values approach isotropy.

Bagherpour (2015) shows the uncertainty as the variance of a random process,
Figure 3.23: Probability distribution of vertical droplet velocity for various droplet size classes at $z/L = 180$. a.) All size classes b.) $d < 25$ c.) $25 \leq d < 40$ d.) $40 \leq d < 60$ e.) $d \geq 60$

limited by number of measurement samples:

Table 3.5: Estimator variances multiplied by N (Bagherpour, 2015)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Valid for any distribution</th>
<th>Normal assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}$</td>
<td>$\sigma^2$</td>
<td>$\frac{\sigma^2}{\sqrt{n}}$</td>
</tr>
<tr>
<td>$\sqrt{\frac{\sigma^2}{n}}$</td>
<td>$\left[ \frac{\bar{x}^2 - (\bar{x})^2}{4\bar{x}^2} \right]$</td>
<td>$\frac{\sigma^2}{2}$</td>
</tr>
</tbody>
</table>

The average statistical uncertainty of the mean droplet streamwise velocity over all the measurement points can be determined using Tab. 3.5 as 1.3 %, assuming that the data is normal. The average statistical uncertainty of the rms value of the droplet streamwise velocity is determined to be on average equal to 0.65 %, assuming that the distribution is normal. The average statistical uncertainty of the mean
Figure 3.24: Probability distribution of streamwise droplet velocity for various droplet size classes at $z/L = 221$. a.) All size classes b.) $d < 25$ c.) $25 \leq d < 40$ d.) $40 \leq d < 60$ e.) $d \geq 60$

droplet velocity in the vertical direction is equal to 15 % and the uncertainty in the corresponding rms value is 7.5 %. The uncertainty in the mean is higher because the larger diameter droplets have larger settling velocities and the uncertainty term is for the whole population, regardless of position, velocity or size class. Bagherpour (2015) also describes the uncertainty of the PDI instrumentation as noted by the manufacturer, Artium:

Table 3.6: Phase Doppler interferometry uncertainty (Bagherpour, 2015)

<table>
<thead>
<tr>
<th></th>
<th>Artium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Droplet Size accuracy</td>
<td>$&lt; \pm 0.5 \mu m$</td>
</tr>
<tr>
<td>Droplet Velocity accuracy</td>
<td>$&lt; \pm 1%$</td>
</tr>
<tr>
<td>Volume Flux accuracy</td>
<td>$&lt; \pm 15%$</td>
</tr>
</tbody>
</table>
Figure 3.25: Probability distribution of vertical droplet velocity for various droplet size classes at z/L = 221.  a.) All size classes b.) d < 25 c.) 25 ≤ d < 40 d.) 40 ≤ d < 60 e.) d ≥ 60
Figure 3.26: Probability distribution of droplet velocity for various droplet size classes at $z/L = 328$. a.) All size classes b.) $d < 25$ c.) $25 \leq d < 40$ d.) $40 \leq d < 60$ e.) $d \geq 60$
Figure 3.27: Probability distribution of vertical droplet velocity for various droplet size classes at \(z/L = 328\). a.) All size classes b.) \(d < 25\) c.) \(25 \leq d < 40\) d.) \(40 \leq d < 60\) e.) \(d \geq 60\)
3.5 Droplet Velocity Profiles

Fig. 3.28 shows the deficit of the droplet velocity relative to the gas phase in the spray plume. Each panel is a vertical profile at horizontal positions across the flow. It can be seen that the deficit is small, but significant at the nearest position to the grid but diminishes with downstream distance. Fig. 3.29 shows the corresponding streamwise turbulence profiles. The turbulence decreases with downstream distance consistent with the gas phase turbulence.

Figure 3.28: Percentage of velocity deficit for droplets of all size classes at various cross-stream positions for each y-z plane. (a.) x/L = 7  b.) x/L = 4  c.) x/L = 0  d.) x/L = -4  e.) x/L = -8)

Fig. 3.29 normalizes the rms droplet velocity with mean dispersed phase velocity. One can note a minimum of droplet turbulence intensity around 1% between y/L = -5 and 5 for all x positions and z/L ≥ 180. For z/L = 57, increased turbulence near
the lower boundary is seen in Fig. 3.29 for those corresponding positions.

Figure 3.29: Droplet turbulence intensity in z-direction for droplets of all size classes at various cross-stream positions for each y-z plane.
(a.) x/L = 7 b.) x/L = 4 c.) x/L = 0 d.) x/L = -4 e.) x/L = -8)

In Fig. 3.29 it appears as though droplet velocity variance tends to increase between the centerline and the outermost point measured. The geometry of the plume most likely coincides with this same area, hence the higher turbulence in the y/L = 4-8 area. These numbers correspond to the gas-phase turbulence measurements although there appears to be some higher turbulence levels near the edges of the measurement region.
Figure 3.30: Droplet turbulence intensity in y-direction for droplets of all size classes at various cross-stream positions for each y-z plane.
(a.) x/L = 7 b.) x/L = 4 c.) x/L = 0 d.) x/L = -4 e.) x/L = -8)
3.6 Concentration

The mean volume concentration contours are shown in Figs. 3.31 to 3.34 for the different downstream positions. The measurement grid, as defined in Fig. 3.1, is a 5 x 15 grid; therefore the resolution is not fine enough to draw definitive conclusion about the spray behaviour from the contour plot. They can, however, be used as a tool for visualizing the flow. The separation of the different droplet sizes can be observed as well as the distortion of the contour profile. This distortion increases with downstream distance and becomes skewed towards the geometric centerline. Note

Figure 3.31: Contour of mean volume concentration for all droplet size classes at \( z/L = 57 \).
Figure 3.32: Contour of mean volume concentration for all droplet size classes at \( z/L = 180 \).

Figure 3.33: Contour of mean volume concentration for all droplet size classes at \( z/L = 221 \).
Fig. 3.34: Contour of mean volume concentration for all droplet size classes at $z/L = 328$.

Fig. 3.35 shows profiles of mean volume concentration for all size classes. One can see that the maximum concentration is behind the nozzle at all downstream locations and that the maximum concentration decreases as the position moves horizontally away from the centerline. Fig. 3.36 shows the small droplet concentration profiles across the various x planes is typical for all size classes. These profiles are not symmetrical about $x/L = 0$. This suggests that the centerline of the flow lies somewhere between $x/L = 0$ and $x/L = -4$. It is also of interest to note that as $z/L$ moves away from the grid/nozzle the droplets diffuse outwards. Fig. 3.37 for the medium droplets shows increases in downstream concentration in areas which had seen lower concentration. This occurs because of the sinking droplets. The volume concentration profiles for the large and extra large droplets ($40 \leq d < 60$ and $d \geq 60$) shown in Figs. 3.38 and 3.39 appear to affected less by the $z$ position than the volume concentration profiles closer to the grid. There is loss of statistical significance in the profiles for the extra large droplets as fewer droplets were available in these
regions. The concentration for all size classes seen in Fig. 3.35 looks very similar to the behaviour of the medium to large droplets.
Figure 3.36: Mean volume concentration for small droplets. (a.) x/L = 7  b.) x/L = 4  c.) x/L = 0  d.) x/L = -4  e.) x/L = -8)
Figure 3.37: Mean volume concentration for medium droplets. (a.) x/L = 7 b.) x/L = 4 c.) x/L = 0 d.) x/L = -4 e.) x/L = -8)
Figure 3.38: Mean volume concentration for large droplets. (a.) x/L = 7 b.) x/L = 4 c.) x/L = 0 d.) x/L = -4 e.) x/L = -8)
Figure 3.39: Mean volume concentration for extra large droplets. (a.) $x/L = 7$ b.) $x/L = 4$ c.) $x/L = 0$ d.) $x/L = -4$ e.) $x/L = -8$)
The volume flux is calculated by Artium’s proprietary software, AIMS. It is only capable of measuring the mean volume flux for a given measurement position. This causes some difficulty in calculating instantaneous concentration as well as correlations like $c_v v_z$ and $c_v v_y$. That is why Eq. 1.22 must be used to calculate the correlation between fluctuation concentration and velocity.

The mean volume flux for various sizes classes in each data record can be seen in Figs. D.2 - D.1. The behaviour is almost identical to that which is found in Figs. 3.35 - 3.39. The mean concentration was determined by multiplying the flux by the Favre average velocity, according to Eq. 2.11, which is a scalar, so it is to be expected that the behaviour would largely be unchanged with the exception of a stretch factor proportional to the Favre average velocity.

### 3.7 Correlations of Droplet Velocity and Size

This section describes the relationships between the droplet sizes and both streamwise velocity and vertical velocity. Figure 3.40 provides the measurements of droplet size and velocity at $z/L = 57$, the closest position to the grid. Each subplot corresponds to a vertical elevation. There clearly is a droplet velocity shape between subplots which corresponds to the mean concentration shown in Fig. 3.35 for $x/L = 0$. The droplet sizes and velocities are also symmetric between top and bottom. A wake is not apparent in the absence of spray. When observing the upper and lower half of Fig. 3.40, the formation of two size groups can be seen, which does not exist near the center of the flow. This suggests that the larger droplets originate around the edge of the spray. There does not appear to be a correlation between droplet size and streamwise velocity.

Looking at data at further positions downstream at $z/L = 180, 221$ and $328$, several observations can be made. There is a smaller range of droplets sizes and the
magnitude of streamwise velocity fluctuations for droplets of all sizes is diminished. This is especially true of the largest droplets. The large droplets, that were present at the top of the flow, are gradually diminished with increased distance downstream. At intermediate distances this corresponds to an increase of large droplets at the center and lower levels. At the furthest downstream positions the large droplets have almost all migrated to the bottom levels. The diameter which divides the two populations tends to increase as the elevation decreases. There are few recorded droplets at these separation points and they also have lower turbulence levels.
Figure 3.40: Droplet size correlation with streamwise velocity on center line for $z/L = 57$. (a.) $y/L = 14$, b.) $y/L = 10$, c.) $y/L = 6$, d.) $y/L = 2$, e.) $y/L = 0$, f.) $y/L = -2$, g.) $y/L = -6$, h.) $y/L = -10$)
Figure 3.41: Droplet size correlation with streamwise velocity on center line for z/L = 180. (a.) y/L = 14, b.) y/L = 10, c.) y/L = 6, d.) y/L = 2, e.) y/L = 0, f.) y/L = -4, g.) y/L = -10, h.) y/L = -18)
Figure 3.42: Droplet size correlation with streamwise velocity on center line for \( z/L = 221 \). (a.) \( y/L = 14 \), b.) \( y/L = 10 \), c.) \( y/L = 6 \), d.) \( y/L = 2 \), e.) \( y/L = 0 \), f.) \( y/L = -4 \), g.) \( y/L = -10 \), h.) \( y/L = -18 \)
Figure 3.43: Droplet size correlation with streamwise velocity on center line of downstream location, $z/L = 328$. (a.) $y/L = 12$, b.) $y/L = 8$, c.) $y/L = 4$, d.) $y/L = 0$, e.) $y/L = -2$, f.) $y/L = -6$, g.) $y/L = -10$, h.) $y/L = -18$)
3.8 Correlations of Concentration and Velocity

The measured correlations between droplet velocity components and concentration, \( v_z v_y, c_v v_z, \) and \( c_v v_y \) were determined from Eq. 1.22. Results are shown in Figure 3.48 to Figure 3.51. For Figure 3.48 and 3.50 each subplot is a vertical profile at a different horizontal position across the flow. The data were normalized by \( C_{v0} v'_z \), where \( C_{v0} \) is the mean centerline volume concentration, and values of \( v'_z \) and \( v'_y \) used can be found in Tab. 3.3 and 3.4. Figure 3.48 shows there are some negative correlations near the centerline of the flow, in the region of maximum concentration, otherwise there is minimal correlation between the concentration and streamwise velocity.

These results show that where there is a high concentration of droplets, seen in Figs. 3.35-3.39, there is a negative correlation effect between the local fluctuating streamwise velocity of those droplets and the local concentration fluctuation. This could be due to increased droplets interactions that would exist in the spray plume or could be explained by the gradient of volume concentration or droplet velocity due to the wake from the grid that is only apparent near the center of the cross section.

Fig. 3.49 presents the same data as Figure 3.48 along the centerline, but separated for each size class. It is important to note that in this case the data is normalized by the centerpoint concentration for the respective size class and the vertical centerline rms values. The correlations are, again, small everywhere, but in this figure it is clearer to see that large droplets close to the nozzle have a higher correlation where \( y < 0 \) and and the furthest downstream positions. The turbulence is most prominent near the nozzle and gets diffused through the test section as the plume travels downstream. The concentration behaves in a similar fashion; the droplets are concentrated near the nozzle but then begin to diffuse outward as the spray plume travels downstream. There are shifts in the signs above and below the centerline.

The data for the vertical velocity correlation with the concentration is normalized by the centerpoint concentration as before, but the root-mean squared velocity is
found in Tab. 3.4. Figs. 3.50 and 3.51 show more locations with stronger correlations. It can be seen that $z/L = 328$ shows the strongest correlations for all of the $x/L$ positions. It is interesting that the correlation on the centerline is strongest for the data closest to the grid and then trails off as the plume travels downstream. The inverse becomes true as the measurement location moves away from the centerline. For example, in Fig. 3.50d.) the strongest correlations are present at the furthest downstream position. This suggests the flux in the y-direction increases towards the walls as the plume moves downstream. Fig. 3.51 shows that the droplets close to the nozzle have a fairly symmetric profile along the centerline. These correlations are much stronger because the normalized parameters are only for the specific sizes, but it allows for a clearer depiction of the correlation profile. The large ($40 \leq d < 60$) droplets show the strongest vertical turbulent flux that increases as the spray plume travels downstream. The range of strong correlation increases as the plume moves downstream, which is potentially caused from the turbulent transport mixed with gravitational effects.
3.9 Droplet Shear Stresses

Figure 3.52 shows turbulent shear stress of the droplet velocities normalized by the squared mean streamwise velocity. The shear stress is strongest in the wake but are relatively weak elsewhere. In the wake, close to the nozzle (third subplot down), it appears that there is a positive shear stress and a sign change across the center of the flow, to counteract it near the edges of the spray plume. Further away from the centerline the shear stress diminishes. The opposite can be seen to be true away from the centerline, where it is strongest near the edges. The same graph is created on the centerline with each subplot showing different $z/L$ positions and each data series demonstrating the behaviour of the different size classes. In accordance with the high turbulence intensity near the bottom of the test section, the turbulence shear stress is higher in that location for all the droplet sizes. The turbulent shear stress can be seen to be strongest for the smallest size class droplets (subplot a.) and an order of magnitude smaller for the other size classes. The shear stress is also more significant where $y/L < 0$. 
Figure 3.44: Droplet size correlation with vertical velocity on center line for $z/L = 57$. (a.) $y/L = 14$, (b.) $y/L = 10$, (c.) $y/L = 6$, (d.) $y/L = 2$, (e.) $y/L = 0$, (f.) $y/L = -2$, (g.) $y/L = -6$, (h.) $y/L = -10$)
Figure 3.45: Droplet size correlation with vertical velocity on center line for $z/L = 180$. (a.) $y/L = 14$, (b.) $y/L = 10$, (c.) $y/L = 6$, (d.) $y/L = 2$, (e.) $y/L = 0$, (f.) $y/L = -4$, (g.) $y/L = -10$, (h.) $y/L = -18$)
Figure 3.46: Droplet size correlation with vertical velocity on center line for $z/L = 221$. (a.) $y/L = 14$, (b.) $y/L = 10$, (c.) $y/L = 6$, (d.) $y/L = 2$, (e.) $y/L = 0$, (f.) $y/L = -4$, (g.) $y/L = -10$, (h.) $y/L = -18$)
Figure 3.47: Droplet size correlation with vertical velocity on center line for z/L = 328. (a.) y/L = 12, b.) y/L = 8, c.) y/L = 4, d.) y/L = 0, e.) y/L = -2, f.) y/L = -6, g.) y/L = -10, h.) y/L = -18)
Figure 3.48: Correlation of volume concentration and streamwise droplet velocity for all size classes normalized by centerline concentration and \( v'_z \). (a.) \( x/L = 7 \) b.) \( x/L = 4 \) c.) \( x/L = 0 \) d.) \( x/L = -4 \) e.) \( x/L = -8 \)
Figure 3.49: Correlation of volume concentration and streamwise droplet velocity for various size classes on centerline normalized by centerpoint concentration and $v_z'$. (a.) $z/L = 57$ b.) $z/L = 180$ c.) $z/L = 221$ d.) $z/L = 328$
Figure 3.50: Correlation of volume concentration and vertical droplet velocity for all size classes normalized by centerpoint concentration and $v'_z$. (a.) $x/L = 7$ b.) $x/L = 4$ c.) $x/L = 0$ d.) $x/L = -4$ e.) $x/L = -8$)
Figure 3.51: Correlation of volume concentration and vertical droplet velocity for various size classes on centerline normalized by centerline concentration and $v_z'$. (a.) $z/L = 57$ b.) $z/L = 180$ c.) $z/L = 221$ d.) $z/L = 328$
Figure 3.52: Normalized turbulent shear stress for all droplet size classes. (a.) x/L = 7 b.) x/L = 4 c.) x/L = 0 d.) x/L = -4 e.) x/L = -8)
Figure 3.53: Normalized turbulent shear stress for various droplet size classes on centerline y-z plane. (a.) $z/L = 57$ b.) $z/L = 180$ c.) $z/L = 221$ d.) $z/L = 328$)
The trends of the shear correlation coefficients, $\frac{v_z v_y}{v_x' v_y'}$, are shown in Figs. 3.54 and 3.55. The correlation must lie between -1 and 1. In sheared turbulence near the edge of the spray, the correlation coefficient typically has values of ±0.5. It can been that for the position closest to the grid the correlations had a smaller range, and they were more correlated as the measurement position moved from the centerline. In addition, stronger correlations overall were observed as the measurement position migrated downstream.

Figure 3.54: Droplet velocity shear correlation coefficient for all droplet size classes. (a.) $x/L = 7$ b.) $x/L = 4$ c.) $x/L = 0$ d.) $x/L = -4$ e.) $x/L = -8$)
Correlation coefficients for different size classes are shown in Fig. 3.55. This figure captures only the data on the vertical centerline and each subplot corresponds to the different downstream position. The correlations are similar for all size classes at these positions.

Figure 3.55: Droplet velocity shear correlation coefficient for various droplet size classes on centerline y-z plane. (a.) z/L = 57  b.) z/L = 180  c.) z/L = 221  d.) z/L = 328)
Chapter 4

Discussions

4.1 Sink Rate

The sink rates for droplets of each size class in still air determined from Eq. 2.2 and shown in Table 4.1.

Table 4.1: Droplet sink rates in still air for various size classes.

<table>
<thead>
<tr>
<th>Size Ranges (µm)</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>Extra Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>d &lt; 25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 ≤ d &lt; 40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 ≤ d &lt; 60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d ≥ 60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Diameter (µm)</td>
<td>20</td>
<td>33</td>
<td>49</td>
<td>93</td>
</tr>
<tr>
<td>Re_D</td>
<td>0.0162</td>
<td>0.0683</td>
<td>0.2183</td>
<td>1.3621</td>
</tr>
<tr>
<td>V_{sink} (m/s)</td>
<td>0.0121</td>
<td>0.0314</td>
<td>0.0677</td>
<td>0.2198</td>
</tr>
</tbody>
</table>

The tabulated results were compared with the vertical velocity probability distributions seen in Figures 3.21-3.27 to observe the impact of the droplet turbulence on the sink rate for the various droplet sizes. From the vertical velocity populations it can be seen that the small, medium and large droplets within each x-y plane along the length of the test section exhibit a mean sink rate of approximately 0.2 m/s.
This is the expected sink rate for an extra large droplet in still air. The extra large droplets are observed to travel downwards significantly faster at speeds of around 0.5-0.6 m/s. The sink rate does not change significantly with increasing downstream distance. These results suggest that the large droplets interact with the smaller droplets through the gas phase to increase the sink rate of smaller droplets.

4.2 Separation of Droplet Size Classes

The majority of droplets were less than 60 µm with the frequency of occurrence of droplets smaller than 60 µm of around 60%. The distribution is approximately log-normal which is typical for polydisperse sprays with the majority of the droplets corresponding to the small, medium and large droplets seen in Tab. 3.2. The average Stokes number for all size classes, based on the scale of the energy-containing eddies, ranges from $0.09 < St_e < 1.7$ which spans the desired range of 0.01-1.99 laid out in the objectives.

The migration of the different size classes due to the effects of gravity can be observed in Fig. 3.8. Near the nozzle the droplet size profile is symmetric about the origin of the y-axis. The larger droplets are at the edges of the plume and the center of the spray plume contains a high concentration of small droplets. As the spray travels downstream to $z/L = 180$, the vertical symmetry is lost with more of the large and extra large droplets located at the center and near the bottom of the spray. The data series $x/L = -4$ and $x/L = -8$ show some larger droplets near the center of the flow which could potentially be due to larger droplets falling from the upstream location. At the next location, $z/L = 221$ shown in Fig. 3.10 the trend continues with fewer extra large droplets near the top of the spray plume and more near the bottom. At the sides of the spray, $x/L = -4$ and -8 the larger droplets are no longer present. Figs. 3.9 and 3.10 also had data measurement sites at lower positions on the y-axis.
because of the large quantity of droplets in these regions. Finally, Figure 3.11, z/L = 328 shows that there are no longer any droplets that are greater than 60 \( \mu m \) above the centerline.

It is also important to note that the larger droplets make up more of the overall volume and since the volume mean diameter, or \( d_{30} \) is determined from

\[
D_{30} = \left[ \frac{\sum_{i=1}^{\infty} n_i D_i^3}{\sum_{i=1}^{\infty} n_i} \right]^{\frac{1}{3}}
\]  

(4.1)
large droplets are disproportionally represented. The effects can be seen clearly in Fig. 3.7, showing both the percentage fraction of droplet in terms of both number and volume. That is one of the reasons why it is important to also consider the size distribution of the droplets. Similar patterns of large droplet migration may be observed in the probability distribution results where the effects of separation are more prominent. Fig. 3.12, zL = 57 shows how the size distribution seems to indicate that there are two populations which are separated. One of these populations is constituted of the small, medium and large size classes and the other is made up of only extra large droplets. At this position there are, again, mostly small droplets near the centerline of the spray plume, which could also be seen in the volume mean diameter. At z/L = 180 (in Fig. 3.13) there are some larger droplets that could have fallen from above and the population at y/L = 14 fills out with large droplets in the range of 40-60 \( \mu m \). It can also be seen that the droplets which were present at y/L= 10 and z/L = 57 appear to fall to positions y/L = 6 at z/L = 180. In between z/L = 180 and z/L = 221 the small and medium droplet size classes both have increased probabilities of occurrence, whereas the larger droplets have again decreased in probability of occurrence at y/L = 14. At the furthermost downstream position there are small, medium and large droplets at all positions, but the extra large droplets over 100 \( \mu m \) have fallen down to the lowest measurements positions.
The vertical velocity distributions seen in Figs. 3.21, 3.23, 3.25 and 3.27 show that the mean vertical velocity is negative with the largest (-0.5 m/s) value for the extra large size class, but in the vicinity of -0.2 m/s for the other size classes. This reinforces the observation that the largest droplets travel downwards with a sink rate that is higher than that of the other size classes producing the droplet size separation that is observed.

The concentration profiles provide another method of visualizing the flow and the droplet separation more directly. Figure 3.35 shows that for all size classes the concentration profile is not symmetric about the origin. The different data series show the various z stations of measurement where it can be seen that the point of maximum concentration, aligned with the nozzle, shifts only slightly between the data series. As expected for the small, medium and large size classes, with smaller Stokes numbers, the location of where the maximum concentration occurs does not migrate as the flow travels downstream because these classes experience transport from the turbulent eddies that obscure the forces of gravity. The concentration of the large size class, Fig. 3.38, shows that the point of maximum concentration is migrating and getting wider. The separation of the droplet sizes is even more evident in the figure showing the extra large size class, 3.39, where the concentration becomes flatter and wider with downward skewness. The effect which was seen in the smaller droplets, where they were diffused outward as the plume moves downstream by turbulent convection, is not seen in Figure 3.35, because the behaviour is dominated by the behaviour of the larger size classes. The droplets with larger Stokes number are unaffected by the z-position and generally remain on their trajectories, dominated by their inertial forces, unless they are large enough to have non-negligible sink rate.

The correlations between droplet diameter and streamwise velocity are observed through visual inspection of Figures 3.40 to 3.40 which show the individual measurements of the droplet diameter and streamwise velocities along the centerline of the
spray plume. There appears to be only a slight correlation between droplet size and streamwise velocity noticeable for the larger droplets in Figure 3.43 for $z/L = 328$. Noting the vertical velocity components, Figs. 3.44 to 3.47 show that there is a strong negative correlation between the droplet diameter and the vertical velocity of the droplet. The larger droplets fall at a higher sink rate, which was confirmed in the probability densities of the vertical velocity along the centerline.

### 4.3 Droplet Turbulence

The effects of gas-phase turbulence on droplet velocity distributions (Figs. 3.20-3.26) in the wind tunnel can be seen on the different size classes as the spray plume moves downstream in the wind tunnel. The rms droplet velocities in the wind tunnel in both the transverse direction and the longitudinal direction are summarized in Tables 3.3 and 3.4, respectively. From Tables 3.3 and 3.4 it can be seen that there is evidence of droplet turbulence kinetic energy decay with downstream distance as $z/L$ increases. Similar to the gas phase turbulence (Figures 3.5-3.6) the rms droplet velocity has decreased by around 50% of what it was close to grid for all of the size classes. However, the medium size class has the smallest turbulence level, especially at $z/L = 180$. The extra large size class has the largest rms droplet velocity. The was a surprise given that the large size class with an average Stokes’ number of 1.7 was expected to be less influenced by gas-phase turbulence. Similar behaviour is seen in Table 3.4 for the transverse/vertical fluctuations, but the magnitudes are reduced. An explanation could be the disproportional influence of the interarrival time in the Reynolds averaging regime. The large and extra large droplets may have had bursts of short interarrival times due to droplets which coalesced on the grid in a region with an otherwise long gap between droplets which could adversely influence the fluctuating component of the velocity in both the streamwise and vertical directions.
A probability curve was fitted to the distribution of the velocity seen in Figs. 3.20 to 3.26 from the following equation for random variable $x$ with mean, $\mu$ and standard deviation, $\sigma$:

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2}$$

(4.2)

where all the statistical parameters considered are Reynolds averaged. Considering all positions the observed streamwise velocity probability density function in Figure 3.16 does not conform to the expected Gaussian. Near the grid it is flatter and skewed to the right. As the spray plume moves downstream the velocities become more Gaussian. Figure 3.24 at $z/L = 221$ shows the probability density is almost Gaussian with the exception of the largest droplets. By the final downstream position, Fig. 3.26 shows that all of the size classes have become Gaussian. The larger size classes take longer to respond to these effects.

The vertical velocity distributions seen in Figs. 3.21, 3.23, 3.25 and 3.27 show that the turbulence level in the vertical direction also decreases as the distance downstream increases. These distributions are approximately normally distributed across each x-y plane throughout the entire flow, unlike the streamwise velocity. The turbulence intensities in both the transverse and longitudinal planes are relatively close in magnitude (Figs. 3.30 and 3.29).

The droplet shear stresses are presented in Figures 3.52 and 3.53. The information in Figure 3.52 is for all size classes at all positions within the flow. It is notable that the regions of higher shear stress correlate with regions of high turbulence energy, seen in Figs. 3.29 and 3.30 as close to the grid (positions $z/L = 57$ and 180) and near the upper and lower walls. Figure 3.53 shows the normalized turbulent shear stresses along the centerline for different size classes. Subplot a.) of Fig. 3.53 shows that the shear stress present among the smallest droplets constitutes the majority of the turbulent shear stress in the flow. Hence, the smaller droplets are easily sheared by the flow. A slight slope can be seen for the other size classes, subplots b.)-d.).

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This is an indication that there is a slight shear in the flow as a result of interactions between the droplets and the gas phase.

Figures 3.54 and 3.55 show the correlation coefficient for the two velocities measured in this experiment. All correlations regardless of size class range from -0.5 to 0.5, which is typical of sheared turbulence. The correlations follow the same general trend of the small droplet shear stress, in that they show a change in signs near the nozzle location. This trend is due to the outwards migration of the droplets, due to gravity and turbulent convection as they move downstream and increase in streamwise velocity. The correlations along the centerline for different size classes as the droplets travel downstream is seen in Figure 3.55.

4.4 Correlation Effects

Figure 3.48 shows the correlation of the droplet velocity with the volume concentration at each measured point in flow for all droplet size classes. The correlations are minimal near the edges of the spray plume, as seen in subplots 3.48a.) and 3.48d.). Near the center of the flow, in subplots 3.48b.)-3.48d.) the correlations are stronger. This region is where the majority of the droplets are located, an indication that the droplets are transported down the gradient of concentration by the random droplet motions. At z/L = 328, a sign change occurs for x/L = 4 and x/L = 0 near y/L = -2 with a minimum value near y/L = -4. The data cross the x-axis slightly below the nozzle location, followed by maximal value near y/L = 0 for x/L = 0. These data suggest that there is a positive change in the velocity as a result of a change in concentration above the centerline and a negative change below the centerline in the left half of the spray plume (x/L < 0). The correlation stays positive for x/L > 0. The uneven correlation field may be attributed to the shape of the droplet sheet created by the nozzle itself.
Figure 3.49 represents a centerline snapshot of the data in Fig. 3.48 across each size class. The notable features shown are the overall higher correlation near the grid, where $z/L = 57$, the sign change of the correlation across the centerpoint, and the higher correlation among the larger droplets. There is a stronger negative correlation below $y/L = 0$ for the larger droplets at $z/L = 328$ which are now present in the lower regions of the spray plume because of gravity.

The vertical velocity is correlated strongly with the concentration near the centerline where $z/L = 57$, seen in Figure 3.50. The correlation increases for the downstream locations away from $x/L = 0$. The same behaviour was seen in the concentration profiles. Figure 3.51 shows an increased correlations for larger droplets close to the nozzle. The largest droplet size classes show two distinct peaks which merge and flatten out as $z/L$ increases. The peaks are not symmetric about the origin. As the plume travels downstream the maximum correlations are found at different locations.
Chapter 5

Summary and Conclusions

The research was successful in that the design of the experiment allowed for a small wake with and without a grid in place on the nozzle of the wind tunnel with the spray assembly set up inside of the settling chamber. The wake was minimized due to acceleration of the gas-phase that occurred within contraction of the wind tunnel and the presence of the grid.

The size range of the droplets which were present in the flow matched up with the desired range in the objectives of $10 - 140 \ \mu m$. The Stokes number for the different droplet size classes also corresponded favorably to the range stated in the objectives of $0.01 \leq St_e \leq 1.99$, although there were fewer small droplets, $d < 25\mu m$ (12% of the observed total number of droplets) and a large amount of droplets between $25 - 60 \ \mu m$. There was good agreement between the dimensionless sink rate, $\frac{\eta_d}{v}$, for the atmospheric boundary layer and the grid turbulence. The Stokes numbers based on the Kolmogorov microscales agreed as well (See Tab. 1.1. The Stokes number based on the LES grid spacing for the atmospheric boundary layer and the grid spacing for the wind tunnel experiment, $St_e$ show some slight overlap, but it was difficult to get all of the parameters to match. The Stokes number for the largest droplets in the atmospheric turbulence, $250\mu m$, matches with the smallest droplets found in the grid.
turbulence experiment. A notable feature of the flow was the migration of the larger droplets to the bottom of the measurement area due to the presence of gravity. The effects of the migration were mostly visible in droplets with higher Stokes numbers, \( St_e > 1 \). The turbulence within the flow prevented the smaller droplets from falling as quickly as the larger droplets although an increase in sink rate was seen in comparison with a droplet in still air. The largest droplet size class were seen to fall around two times faster than all other size classes.

The PDI was able to successfully measure two components of velocity for all of the droplets as well as corresponding droplet diameter. The Artium software package made it possible to obtain the volume flux of the droplets and probe volume concentration, from which the volume concentration was calculated. The concentration profiles showed that the majority of the volume was concentrated near the nozzle axis. This region was made up of almost entirely small droplets with diameters less than 40 \( \mu m \). The larger droplets were on the edges of the spray plume as seen in the figures of the volume mean diameter due to the nature of the inpingement type nozzle which was used. As the plume travelled downstream an increase in concentration was seen near the top and bottom of the measurement area, indicating droplet diffusion. This effect was seen mostly within the smaller droplets, but could also be observed in the large droplets. Whereas turbulent diffusion was the main mechanism for the transport of the smaller droplets, gravity was the main mechanism in transporting the droplets with diameters over 60 \( \mu m \). The separation of the droplets was also highlighted in the correlations between droplet diameter and vertical velocity. The larger droplets where showing large negative vertical velocity, i.e. larger sink rates.

The turbulence of the droplet phase decreased in intensity with increased downstream distance because the exchange of energy between the two phases within the flow. The droplets dampened the turbulent energy in the streamwise direction within the flow as they required energy to be accelerated to match the mean gas-phase veloc-
ity. The corresponding shapes of the streamwise velocity probability densities were also less Gaussian. The vertical velocity maintained a Gaussian profile at all downstream measurement locations and the turbulence decayed similarly to the streamwise velocity. The larger droplets showed some unexpected turbulence at the most downstream location which could perhaps be due to the Reynolds regime and the large interarrival times between droplets.

The flow was nearly isotropic at downstream positions as the magnitudes of the turbulence in the streamwise and vertical directions were similar in magnitude and homogeneity was achieved as there was no change in velocity gradient across the cross-section.
Chapter 6

Future Work

Further exploration of homogeneous isotropic turbulence using 3-d Phase Doppler Interferometry in a controlled setting would be helpful to ascertain the other velocity and concentrations correlations which could not be obtained in this research, $v_x v_y$, $v_x^2$, $v_x v_z$ and $c_v v_x$. The concentration profiles in this research showed that the potential centerline of the flow was between $x/L = 0$ and $x/L = -4$, as the concentration profiles were lacking in symmetry in some cases, notably for the profiles of the small droplets. A 3-dimensional investigation may not be necessary assuming symmetry within the measurement area. A more thorough study of the same phenomenon with a higher spatial resolution may perhaps be insightful and prove as a fitting undertaking for a potential PhD candidate.
References


M. W. Reeks. On a kinetic equation for the transport of particles in turbulent flows. 


Euler Equations: 250 Years On.

Appendix A

Instrumentation

A.1 Methodology

A.1.1 Phase Doppler Interferometry

The Artium Technologies PDI 300MD phase doppler interferometer is the chosen measurement device for the purposes of this study. It was chosen because it is able to quantify diameter and three components of velocity simultaneously for a minute measurement volume. However, for the proposed research, only 2 components need to be measured simultaneously due to symmetry in the cross-section of the wind tunnel and the isotropic, homogeneous nature of the turbulence in grid turbulence.

The PDI 300 is a three-component phase Doppler interferometer that uses diode pumped solid-state (DPSS) as the source of illumination. The two pairs of lasers used in the transmitter are: green at 532 nm and blue 473 nm. For the purposes of the proposed research only two pairs of lasers need to be utilized since only two components of velocity must be determined at a time. Each pair of lasers is only able to measure a velocity component. The two pairs of lasers measure velocity components perpendicular to each other, but only the green laser is able to measure the droplet sizes.
The PDI instrument uses the principles of light scattering interferometry to provide reliable performance. The measurement scale used is the wavelength of the light and hence, not as easily degraded. In addition, the method of operation does not necessitate frequent calibration, in fact it has been said only the initial factory calibration will suffice (Artium Technologies Inc., 2009). The PDI system is designed to minimize uncertainty in small droplet measurements by using large optical apertures. The frequency and phase signal processors are able to provide reliable signal detection, mixing and sampling of the signals.

There are a wide variety of possible set-ups for the PDI transmitter and receiver, but it is recommend that the collection angle, i.e. the angle between the transmitter and the receiver, be between 25-40 degrees. This range is optimal because it provides the most intense scattering once a droplet passes through the measurement.
volume formed from the intersection of the two lasers. On top of providing a strong signal it also minimizes reflections of incident light from the beam back towards the transmitter. With the aforementioned collection angle, the ratio between the desired refraction through the droplet to reflection off of the surface becomes 80 to 1 (Artium Technologies Inc., 2009). The set-up for the proposed experiment will be discussed in the Experimental Set-up chapter of this thesis, however, it will be quite similar to Fig. A.1. Note that the transmitter and receiver both must be oriented in a plane orthogonal to the green laser, which measures droplet sizes. The phase Doppler interferometer uses two distinct measurement methods for calculating the diameter of the droplets and the velocity of the droplets. To calculate the velocity of the droplets, a method used in typical laser Doppler Velocimetry scenarios is used.

As a particle passes through the interference fringes created by the two intersecting beams it refracts (or reflects) light onto the photodetector in the receiver (see Fig. A.2) and produces a Doppler burst signal. A moment when a droplet passes through a fringe creates local high intensity light which corresponds to a higher voltage in the signal processor. From elementary kinematics, if the distance between the fringes is known; the fringe spacing, and the time between high intensity light readings (can also expressed as a frequency), then the velocity of the droplet can be determined.
In the equation above $v$ is the velocity of the particle parallel to the direction of the interference fringes, $f_d$ is the Doppler frequency and $\delta$ is the interference fringe spacing. The raw signal from the photodetectors is shifted such that the frequency dynamic range is compressed. This is accomplished via a Bragg cell within the transmitter. The raw signal, $f_r$ is composed of the Doppler frequency, $f_D$ and the shift frequency from the Bragg cell, $f_s$.

$$f_r = f_D + f_s$$  \hspace{1cm} (A.2)

The Doppler frequency is used for determining the velocity of the droplet passing through the measurement volume is determined from first filtering the raw signal with a high pass filter at 20 MHz to remove the pedestal wave arising due to the presence of the droplet in the beam intersection and any other low-frequency noise. This frequency is passed through a frequency mixer in which a local oscillator is combined with the filtered signal to produce a sum and difference.

$$f = (f_r - f_m) + (f_r + f_m)$$  \hspace{1cm} (A.3)

This resulting frequency, $f$ is passed through the adjustable Analog filter within the software package to eliminate the sum frequency, $(f_r + f_m)$, thus the remaining frequency is given as

$$f_t = f_r - f_m$$  \hspace{1cm} (A.4)

A low pass filter is set just above the highest frequency that would present itself in the flow, $f_t$. The purpose of using a frequency shift (from Bragg cell), a mixer frequency and corresponding analog filter is to ensure high resolution and desirable frequencies.
ranges for the processing functions. The Doppler frequency can be determined from

\[ f_t = f_r - f_m \quad \text{(A.5)} \]
\[ f_t = f_D + f_s - f_m \quad \text{(A.6)} \]
\[ f_D = f_t + f_m - f_s \quad \text{(A.7)} \]

The phase Doppler interferometry measurement technique is related to the difference in phase between the two rays passing through a droplet. The phase shift of light scattered from each beam at a specific point in space will vary in proportion to the drop diameter. Fig. A.3 show a schematic of what happens when a droplet is present in the intersection of the two beams. It can be seen that the two rays refract through the spherical medium with a refraction index of, \( m \) and take two different paths since the rays approach the droplet at different angles of incidence. Both rays reach the same point, \( P \) but ultimately undergo a phase shift since the two paths are of different lengths. At point \( P \) where the two rays intersect an interference pattern will be observed, normally with two detectors, which has a wavelength, \( \Lambda \) and phase shift, \( \phi \). A smaller droplet has higher curvature upon the surface which causes large beam divergence and thus the two detectors will be receiving almost identical signals; small phase difference. It follows that a larger droplet has a smaller curvature which
causes less beam divergence and thus a higher phase difference in the far field. Given the fixed photodetector spacing, $S$ and the observed interference wavelength, $\Lambda$, the phase shift can be calculated from

$$\frac{S}{\Lambda} = \frac{\phi}{360^\circ}$$  \hspace{1cm} (A.8)

However, there may potentially be some ambiguity if the fringe spacing is larger than the wavelength between fringes as the phase shift would be greater than $360^\circ$. A three-photodetector receiver was created by Bachalo to address this issue and is what is used in the PDI 300MD. The phase shifts between all three photo-detectors are compared which allows a higher degree of accuracy and partially eliminates previously mentioned ambiguity. The actual diameter of the droplet is calculated from the following equation

$$d = \frac{F\delta}{s\overline{\Lambda}}$$  \hspace{1cm} (A.9)

where $F$ is the focal length of the receiver, $s$ is the sizing slope factor, $\delta$ is the fringe spacing and $\overline{\Lambda}$ is the weighted average of the wavelength, which is determined by the phase shift in the time time domain as

$$\overline{\Lambda} = 360^\circ \left[ \frac{k_{12}S_{12}}{\phi_{12}} + \frac{k_{13}S_{13}}{\phi_{13}} + \frac{k_{23}S_{23}}{\phi_{23}} \right] \left/ \left[ k_{12} + k_{13} + k_{23} \right] \right.$$  \hspace{1cm} (A.10)

with $k$ being an optical constant. The sizing slope is approximated well for refraction as

$$s = \frac{m}{2\sqrt{2(1 + \cos\phi)} \left[ 1 + m^2 - m\sqrt{1 + \cos\phi} \right]}$$  \hspace{1cm} (A.11)

with $\phi$ as the light-scattering detection angle and $m$ as the index of refraction. Eq. A.10 is calculated for all combinations of receiver and transmitter angles through the use of proprietary methods.
A.1.2 Nozzle Specifications

![Table showing PJ Flow Rates and Dimensions](image)

**Standard Materials:** Brass, 303 Stainless Steel and 316 Stainless Steel.

*Spray angle performance varies with pressure. Contact BETE for specific data on critical applications.*
Chosen Nozzle was PJ10 Nozzle because of optimum flow rate of 0.021 gallons per minute at 60 pounds per square inch.

**A.1.3 Hotwire Anemometry**

![Hotwire Anemometer Probe](image)

Figure A.6: Hotwire anemometer probe

A hotwire is an invaluable tool for obtaining accurate time-resolved velocity measurements at a single point and will be very useful for verifying the carrier phase turbulent intensity. There are various types of hotwire anemometers, but the simplest and most accurate is a constant temperature anemometer. As the name implies it functions by altering the current supplied to the hotwire such that the hotwire remains at a constant temperature, i.e. the net heat loss is 0 (heat in and heat out are equal). The main principle of a constant temperature hotwire anemometer is that in order to maintain the heat that is lost to the environment additional heat must be supplied to the hotwire. Assuming a simplified static analysis, the model can be expressed as

\[
\frac{dE}{dt} = 0 \implies W = H
\]

\[
W = H \implies \frac{V^2}{R_w} = hA(T_w - T_a)
\]

\[
\frac{V^2}{R_w} = \frac{Nu_kA(T_w - T_a)}{d}
\]

(A.12)

where \(E\) is the thermal energy stored in the wire, \(W\) is the power generated by Joule heating, \(H\) is the heat transferred to surroundings, \(h\) is the film coefficient of heat
transfer, $A$ is the heat transfer area, $d$ is the wire diameter, $k_f$ is the heat conductivity and $Nu$ is the Nusselt number. In a forced convection regime, i.e. $Re > Gr^{1/3}$ (0.02 in air) and $Re<140$, noting that Reynold’s number is a function of velocity yields:

$$Nu = A_1 + B_1 Re^n = A_2 + B_2 U^n \quad \text{(A.13)}$$

Substituting the Nusselt number equation, Eq. A.13, into thermal energy balance, Eq. A.12 above, leads to the following equation:

$$\frac{V^2}{R_w} = E^2 = (T_w - T_a)(A + BU^n)$$

$$E^2 = A + BU^n \quad \text{(A.14)}$$

Note that the physical parameters like $Re$, $d$ and $k_f$ get absorbed into new constants $A$ and $B$. Equation A.14 is called King’s Law and is a non-linear equation. It can be linearized and expressed as a polynomial, usually a fourth-order.

$$U = f(E) = k_1 + k_2 E + k_3 E^2 + k_4 E^3 + k_5 E^4 \quad \text{(A.15)}$$

The voltage data from the hotwire bridge can be plotted against the velocity obtained from the pressure transducer to create a calibration curve. The hotwire is placed in the center of the wind tunnel using an adjustable rod which can be moved vertically. A traverse mechanism is used to move laterally and a linear bearing is used to move longitudinally along the length of the wind tunnel.
Appendix B

3D Calculations

Figure B.1: Schematic showing the PDI arrangement and two orientations obtained by swapping receiver and transmitter positions
Figure B.1 shows how two possible configurations for the transmitter and receiver are able to yield two distinct pairs of velocity components. It is important to note that the view in Fig. B.1 is from the end of the wind tunnel looking inwards. The positive directions for the velocity components which are not parallel with the streamwise direction are shown as blue and red vectors. The yellow dot denotes a velocity which is parallel with the streamwise direction of the wind tunnel. Ideally both positions 1 and 2 will yield the same velocity in the streamwise direction as the transmitters are in the same plane, but there will undoubtedly be small amounts of misalignment. The two components, $\vec{V}_1$ and $\vec{V}_2$ can be converted into resolved components of a resultant $\vec{V}$ by subtracting the projections of $\vec{V}_1$ onto $\vec{V}_2$ from $\vec{V}_2$ and $\vec{V}_2$ onto $\vec{V}_1$ from $\vec{V}_1$. This is expressed mathematically as

$$V_{1,2} = V_1 \cos(\theta_1 - \theta_2)$$  \hspace{1cm} (B.1)

$$V_{2,1} = V_2 \cos(\theta_1 - \theta_2)$$  \hspace{1cm} (B.2)

where $V_{1,2}$ is the projection of $\vec{V}_1$ onto $\vec{V}_2$ and $V_{2,1}$ is the projection of $\vec{V}_2$ onto $\vec{V}_1$. $V_1$ is the magnitude of $\vec{V}_1$ and $V_2$ is the magnitude of $\vec{V}_2$. The portion of $V_1$ projected onto $V_2$ can then be subtracted; same for $V_2$ onto $V_1$. Note that $\vec{V}_{1-2}$ and $\vec{V}_{2-1}$ are in the directions of $\vec{V}_1$ and $\vec{V}_2$, respectively. Eqs. B.1 and B.2 can then be substituted to yield

$$V_{1-2} = V_1 - V_2 \cos(\theta_1 - \theta_2)$$  \hspace{1cm} (B.3)

$$V_{2-1} = V_2 - V_1 \cos(\theta_1 - \theta_2)$$  \hspace{1cm} (B.4)
which the resultant $\vec{V}$ can be determined by adding the components of $V_{1-2}$ and $V_{2-1}$ as follows

$$\vec{V} = (V_{1-2,x} + V_{2-1,x}) \hat{i} + (V_{1-2,y} + V_{2-1,y}) \hat{j}$$

$$\vec{V} = [(V_1 - V_2 \cos(\theta_1 - \theta_2)) \cos \theta_1 + (V_2 - V_1 \cos(\theta_1 - \theta_2)) \cos \theta_2] \hat{i}$$

$$+ [(V_2 - V_1 \cos(\theta_1 - \theta_2)) \sin \theta_1 + (V_2 - V_1 \cos(\theta_1 - \theta_2)) \sin \theta_2] \hat{j} \quad (B.5)$$

This can also potentially be more simply expressed as a system of equations of the form, $\vec{V} = [A]x$, as:

$$\vec{V} = T\vec{V} \quad (B.6)$$

where $T$ is the transformation matrix from the PDI coordinate system to lab frame and $\vec{V}$ is the vector of $V_1$ and $V_2$, $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$. The transformation matrix is defined as follows:

$$T = \begin{bmatrix}
  \cos(\theta_1) + \cos(\theta_2 - \theta_1) \cos \theta_2 & \cos(\theta_2) + \cos(\theta_2 - \theta_1) \cos \theta_1 \\
  \sin(\theta_1) + \cos(\theta_2 - \theta_1) \sin \theta_2 & \sin(\theta_2) + \cos(\theta_2 - \theta_1) \sin \theta_1
\end{bmatrix} \quad (B.7)$$
Appendix C

Pressure and Velocity Effects

C.1 Test Matrix

Table C.1: Test Conditions on centerline at downstream position, z/L = 328 for various pressure and velocities

<table>
<thead>
<tr>
<th>Pressure (psi)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>30</td>
<td>16</td>
</tr>
<tr>
<td>40</td>
<td>15</td>
</tr>
<tr>
<td>50</td>
<td>14</td>
</tr>
<tr>
<td>60</td>
<td>13</td>
</tr>
</tbody>
</table>

Test no. #
Table C.2: Mean volume diameter and Sauter mean of droplets on centerline at down-stream position 3 for various pressure and velocities corresponding to test conditions in Tab. C.1

<table>
<thead>
<tr>
<th>Pressure of container (psi)</th>
<th>Tunnel Velocity (m/s)</th>
<th>Tunnel Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5  10  15  20</td>
<td>5  10  15  20</td>
</tr>
<tr>
<td>30</td>
<td>54.2 41.0 42.8 43.4</td>
<td>61.6 46.2 48.2 48.6</td>
</tr>
<tr>
<td>40</td>
<td>42.8 41.0 45.7 41.9</td>
<td>48.5 47.1 51.6 47.4</td>
</tr>
<tr>
<td>50</td>
<td>49.6 51.5 50.4 46.4</td>
<td>58.0 58.2 57.3 53.1</td>
</tr>
<tr>
<td>60</td>
<td>46.4 45.9 47.3 46.1</td>
<td>53.7 51.5 53.4 52.2</td>
</tr>
</tbody>
</table>

\[ d_{30}(\mu m) \quad d_{32}(\mu m) \]

Tables 3.1 and 3.1 were measured with a constant wind tunnel velocity and pressure and only account for the effects of the turbulence at a single position. It is also of interest to study the effects of the carrier phase velocity and the pressure within the pressure container. It is expected that a higher pressure would cause a higher droplet phase velocity upon exiting from the nozzle would cause the inpingement hammer to break the stream into smaller droplets with a lower Stokes number which would more precisely follow the fluid streamlines. Conversely, a lower pressure would result in larger droplets with a larger Stokes number whose trajectory would tend to be dominated by its initial trajectory. To study these effects alone PDI measurements were conducted at the centerline at z-station, \( z/L = 328 \) downstream of the grid. The various tests can be found in Tab. C.1.

The results of the test in Table C.1 can be seen in Table C.2. It follows from the data that as the velocity decreases the Sauter mean and volume mean diameter of the droplets tend to decrease, but only for smaller pressures. The change was not as significant at 60 psi; the lowest measurement of the volume mean diameter was 45.9 \( \mu m \) where the highest was 47.3 \( \mu m \). That is only a difference of 1.4 \( \mu m \), which is
quite small considering how the 30 psi test had a maximum difference of 13.2 μm, for the volume mean diameter. At a given velocity, as the pressure decreases there was not much of a change in the size of the droplets. However, oddly enough, this was not true for low speeds. The droplets got a lot larger for low wind tunnel speeds. The cause of this phenomenon was observed as testing was occurring. At low wind speeds the droplets were able to conglomerate on the mesh grid. The result of this was coalescence into larger droplets which were then shed into the rest of the spray plume. This aggregation of droplets means that the effects changing the pressure and velocity are confounded with the effect of this phenomenon.
Appendix D

Volume Flux Results

Figure D.1: Mean volume flux for all droplets. (a.) $x/L = 7$ (b.) $x/L = 4$ (c.) $x/L = 0$ (d.) $x/L = -4$ (e.) $x/L = -8$)
Figure D.2: Mean volume flux for small droplets. (a.) $x/L = 7$ b.) $x/L = 4$ c.) $x/L = 0$ d.) $x/L = -4$ e.) $x/L = -8$)

Legend:
- $z/L = 57$
- $z/L = 180$
- $z/L = 221$
- $z/L = 328$
Figure D.3: Mean volume flux for medium droplets. (a.) x/L = 7 (b.) x/L = 4 (c.) x/L = 0 (d.) x/L = -4 (e.) x/L = -8)
Figure D.4: Mean volume flux for large droplets.  
(a.) $x/L = 7$  
(b.) $x/L = 4$  
(c.) $x/L = 0$  
(d.) $x/L = -4$  
(e.) $x/L = -8$  

$z/L = 57$  
$z/L = 180$  
$z/L = 221$  
$z/L = 328$
Figure D.5: Mean volume flux for extra large droplets. (a.) $x/L = 7$  b.) $x/L = 4$ c.) $x/L = 0$  d.) $x/L = -4$ e.) $x/L = -8$)
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