EVALUATION OF PRECISE POINT POSITIONING DERIVED ZENITH
TOTAL DELAYS FROM THE NIGERIAN GNSS REFERENCE NETWORK

by

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ABSTRACT

The Zenith Total Delay (ZTD) from ground-based Global Navigational Satellite System (GNSS) observations is a valuable information source for studying the Earth’s troposphere. Since almost all weather is formed in the troposphere, an analysis of a collection of ZTD time series can provide insight about the behavior of the weather of a place. Several institutions around the world involved in meteorological operations assimilate the ZTD from networks of GNSS continuously operating reference stations (CORS) into Numerical Weather Models (NMW) for better weather forecasting. In Nigeria however, there are no operational GNSS networks used for meteorological purposes. The focus of this thesis is to determine the suitability of the Nigerian GNSS Reference Network (NIGNET) stations for meteorological applications by evaluating the ZTDs obtained from it through precise point positioning (PPP). PPP derived ZTDs from surrounding International GNSS Service (IGS) stations are also included for comparison. These PPP derived ZTDs, spanning from 2011 to 2016, are compared with ZTDs computed from the National Centre for Environmental Prediction reanalysis II (NCEP II) global NWM and from the IGS. A comprehensive time series analysis (least-squares spectral analysis) is performed to determine the spatio-temporal variations of the ZTDs of stations across Nigeria and to evaluate the level of agreement between the three (3) ZTD sources. The comparisons generally show good agreement between the 3 sources with the mean differences lower than 24.2 mm and root mean square errors lower than 45.8 mm. The spectral analyses reveal the various periodic oscillations in the ZTD and how they are influenced by pressure and temperature through the component hydrostatic and wet delays of the ZTD. This research contributes to the characterization of the nature of the
troposphere over Nigeria and affirms the relevance of the NIGNET as a tool for meteorology in Nigeria.
DEDICATION

Dedicated to all those struggling to complete their thesis.
ACKNOWLEDGEMENTS

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<tr>
<td>CORS</td>
<td>Continuously Operating Reference Station(s)</td>
</tr>
<tr>
<td>COSMO-DE</td>
<td>Consortium for Small Scale Modeling-Germany</td>
</tr>
<tr>
<td>DOY</td>
<td>Day of the Year</td>
</tr>
<tr>
<td>ECMWF</td>
<td>European Center for Medium-range Weather Forecasts</td>
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<tr>
<td>EE</td>
<td>Equatorial Easterlies</td>
</tr>
<tr>
<td>ERA-40</td>
<td>ECMWF 40-year Reanalysis</td>
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<td>FTE</td>
<td>Final Troposphere Estimates</td>
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<td>GAPS</td>
<td>GNSS Analysis and Positioning Software</td>
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<td>GMF</td>
<td>Global Mapping Function</td>
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<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
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<td>GPS</td>
<td>Global Positioning System</td>
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<td>IERS</td>
<td>International Earth Rotation and Reference Systems Service</td>
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<td>IGS</td>
<td>International GNSS Service</td>
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<tr>
<td>ITD</td>
<td>Inter-Tropical Discontinuity (ITD)</td>
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<tr>
<td>ITRF</td>
<td>International Terrestrial Reference Frame</td>
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<td>IWV</td>
<td>Integrated Water Vapor</td>
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<td>LSCA</td>
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<td>Least Squares Spectral Analysis</td>
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<tr>
<td>NCEP</td>
<td>National Centres for Environmental Prediction</td>
</tr>
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<td>NCEP II</td>
<td>National Centres for Environmental Prediction reanalysis II</td>
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<tr>
<td>NIGNET</td>
<td>Nigerian GNSS Reference Network</td>
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<td>NMF</td>
<td>Neill Mapping Function</td>
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<td>Acronym</td>
<td>Description</td>
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<td>--------------------------------------------------</td>
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<tr>
<td>NWM</td>
<td>Numerical Weather (Prediction) Model</td>
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<td>PPP</td>
<td>Precise Point Positioning</td>
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<tr>
<td>SD</td>
<td>Slant Delay</td>
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<tr>
<td>TC</td>
<td>Tropical Continental</td>
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<tr>
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<td>Tropical Maritime</td>
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<td>University of New Brunswick</td>
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<td>Vienna Mapping Function</td>
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<td>ZTD</td>
<td>Zenith Tropospheric/Total Delay</td>
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<td>ZWD</td>
<td>Zenith Wet Delay</td>
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1. Introduction

1.1 Background

Global Navigation Satellite Systems (GNSS) signals that propagate through the Earth’s atmosphere are refracted due to the different physical characteristics (refractivity indices) of each layer of the atmosphere (Shrestha, 2003). This refraction affects the signal speed and direction and the resulting delay causes an error in GNSS position and timing determination (Nievinski, 2009). The delay is studied based on two separate parts of the atmosphere: the electrically charged ionosphere and the neutral atmosphere (Mendes, 1999). The neutral atmosphere consists of the troposphere, tropopause, stratosphere, and part of the mesosphere. The delay caused by the troposphere accounts for most of the delays in GNSS signal propagation through the neutral atmosphere (Shrestha, 2003). Tropospheric delays of GNSS signal propagation between satellites and receivers are usually transformed to the zenith direction (directly overhead) of the receiver, making the delay, the zenith total/tropospheric delay (ZTD).

Over the years, processing techniques adopted for GNSS observations have been developed. One such technique is known as precise point positioning (PPP), where precise satellite orbit and clock offset information are used in the processing of observations collected by a single GNSS receiver, typically of dual or more frequency capabilities, to determine the three-dimensional coordinates as well as the receiver clock offset, ambiguities and ZTD (Zumberge et al., 1997). This technique further facilitates the use of GNSS not only for positioning and navigation, but also in applications related
to archaeology, atmospheric, environmental and space sciences and meteorology (Leandro et al., 2007).

In GNSS analysis for accurate positioning and navigation purposes, the estimated ZTD is typically removed. For meteorological applications however, because GNSS signals interact with the constituents of the troposphere as they propagate through, the ZTD serves as a valuable parameter of the troposphere (Ghoddousi-Fard, 2009) that reflects the weather and climatic processes, variations, and atmospheric vertical motions (Jin et al., 2007). When used together with meteorological parameters such as surface pressure and temperature, the Integrated Water Vapor (IWV), which informs about the content of atmospheric water vapor in the troposphere, can be estimated (Bevis et al., 1992). Compared to other atmospheric monitoring techniques, GNSS offers the advantages of having high temporal and spatial resolutions and long-term stability (Jin et al., 2007; Isioye et al., 2015). In recent years, the use of GNSS observations to monitor the atmosphere has increased significantly. Although networks of GNSS continuously operating reference stations (CORS) are typically used for accurate position and timing determination, many nations and agencies utilize these networks to derive ZTD and IWV information to be assimilated into operational weather forecasting models (Yuan et al., 2012; Norazmi et al., 2015).

In Nigeria, there are several ground-based GNSS CORS owned and managed by different institutions and organizations under various initiatives. The Nigerian GNSS Reference Network (NIGNET) CORS is the major ground-based GNSS network in operation in the country, and it is managed by the Office of the Surveyor-General of the
Federation. There are also two stations from the International GNSS Service (IGS); one situated in Nigeria, and the other in Benin Republic, adjoining Nigeria in the south-west. NIGNET was established to create a platform for the use of modern space geodetic techniques for surveying, mapping and land administration and management. NIGNET also serves as the fiducial network that defines and materializes a reference frame based on space-geodetic techniques and it contributes to the African Reference Frame project (Jatau et al., 2010). Through a collection of long-term time series of ZTD from GNSS observations, studies of water vapor distribution on a larger geographical scale can be carried out in the country. All over the world, research, and its findings in the use of GNSS for meteorological purposes have been applied in several global and regional operational Numerical Weather Models (NWM) (de Haan, 2013; Lindskogl, 2017).

GNSS stations have higher spatial coverages, with its observations provided at much higher temporal resolutions than other water vapor observational techniques. Because of this, time series analysis of ZTD observations allows for the investigation of periodic signals which may coincide with seasonal atmospheric phenomena such as weather. Information from such analysis enhances the understanding of the climate in the geographical locations of the GNSS stations and the possibility to forecast extreme weather situations.
1.2 Scopes and Objectives of Thesis

The purpose of this thesis work is to evaluate the PPP estimated ZTDs of the various NIGNET stations from global positioning system (GPS) observations. GPS is a constellation of satellites developed by the United States Department of Defence, and so, is a component of the GNSS. As is done by many agencies across the globe, the focus is to evaluate the quality of the PPP estimated ZTDs by comparisons to ZTDs computed from a global NWM and IGS. Time series spectral analyses of the ZTDs using Least Squares Spectral Analysis (LSSA) and Least Squares Self-Coherency Analysis (LSCA) are also performed to evaluate the short-term periodic weather oscillations that drive the temporal and spatial trends, and to compare these with known weather patterns in the country.

The objectives of this thesis are outlined as follows:

1. Estimation of ZTDs from PPP processed GNSS observations from NIGNET and IGS stations in and around Nigeria.

2. Computation of ZTDs through ray-tracing using parameters from a global NWM for NIGNET and IGS stations, with coordinates computed from PPP.

3. Retrieval of IGS estimated ZTDs from IGS server.

4. Assessment of the GNSS PPP estimated ZTDs through comparison with ray-traced NWM ZTD and IGS computed ZTDs.

5. Time series analysis of short-term variations for 6 years of ZTDs obtained from PPP, NWM and IGS using LSSA.
1.3 Outline of Thesis

Chapter 2 provides background information about the troposphere. Its components and the mathematical equations that represent them are discussed. The various tropospheric models and mapping functions involved in this work, from the various data sources, are also discussed. These include the Vienna and General mapping functions (VMF & GMF respectively) and the Saastamoinen, the VMF1 and the Neill neutral atmospheric delay models. The Nigerian weather and its climatic systems and regions, and a brief description of time series analysis with its application in analyzing the ZTDs are also given.

Chapter 3 gives the general discussion of the sources of ZTD estimates used in this thesis. It includes a description of processes involved with obtaining ZTD estimates from the University of New Brunswick’s (UNB) GNSS Analysis and Positioning Software (GAPS) PPP processing engine, from the IGS, and through ray-tracing from the National Centers for Environmental Prediction’s Reanalysis II (NCEP II) NWM. The processes of the LSSA and the LSCA are also described.

In chapter 4, the data processing is presented. The description of the GNSS data set used is given, with comparisons made between the ZTD estimates from the different sources.

In chapter 5, the results of the LSSA and LSCA for the ZTD time series are presented and discussed.

Conclusions and recommendations are given in chapter 6.
2. Troposphere and Climate

This chapter describes the troposphere, tropospheric delays and its components, and their representation (through models and mapping functions). The relationship between the troposphere and weather and climate is also highlighted, with a description of the weather and climatic patterns in Nigeria. The chapter concludes with a discussion about time series analysis and what it entails.

2.1 Troposphere

The troposphere is the lowest layer of Earth’s atmosphere bounded at its top by the tropopause, which separates the troposphere from the stratosphere, and at its bottom by the earth’s surface. Figure 2.1 (www.esrl.noaa.gov) gives a pictorial representation of the layers of the atmosphere. Depending on the latitude and seasons on the earth, the troposphere extends high up between 7 to 16 km from the earth’s surface. By latitude, the height of the troposphere is lowest over the poles and highest at the equator, and by season, it is lower in winter and higher in summer. The troposphere contains about 75-80% of the mass of the atmosphere, and 99% of water vapor in the atmosphere. Most types of clouds are found and formed in the troposphere, and so, almost all types of precipitation and weather occurs within this layer (Russell, 2010).
At the bottom of the troposphere, the air is warmest but gets colder with increasing altitude. However, air pressure and density decrease with altitude. As shown in Figure 2.2 (GSU, n.d.), the most prevalent gases in the troposphere are nitrogen (78%) and oxygen (21%), with the remaining consisting of argon, water vapor, carbon dioxide, ozone and other constituents. Water vapor plays a major role in regulating air temperature because it absorbs solar energy and thermal radiation from the planet's surface. As sunlight enters the atmosphere, a portion is immediately reflected to space, but the rest penetrates the atmosphere and is absorbed by the earth's surface. The earth then remits this energy back into atmosphere as long-wave radiation. Carbon dioxide and water vapor absorb this energy and emit much of it back towards the earth again. This delicate
exchange of energy between the earth's surface and atmosphere keeps the average global
temperature from changing drastically from year to year.

![Gaseous composition of the troposphere](image)

**Figure 2.2 – Gaseous composition of the troposphere**

Water vapor is a highly variable constituent of the atmosphere, both in space and
time. The main source of atmospheric water vapor is the evaporation of water from
waterbodies and transpiration by plants. The concentration is largest near the surface and
drops to very small values at higher altitudes. On average, the quantity of water vapor
above 10 km is negligible. The water vapor content of the atmosphere is also a function
of the local geographic conditions and meteorological phenomena; its concentration is
very small in the polar regions and in large desert regions, with amounts, of less than 1%
of the volume of the air, but quite significant above tropical rain forests, reaching about
4% of the volume of the air (Mendes, 1999).
The exchange and movement of water between the earth and atmosphere is called the water cycle. The cycle, which occurs in the troposphere, begins as the sun evaporates large amounts of water from the earth's surface and the moisture is transported to other regions by the wind. As air rises, expands, and cools, water vapor condenses, resulting in the development of clouds. Clouds cover large portions of the earth at any given time and vary from fair weather cirrus to towering cumulus clouds. When liquid or solid water particles grow large enough in size, they fall toward the earth as precipitation. The type of precipitation that reaches the ground, be it rain, snow, sleet, or freezing rain, depends upon the temperature of the air through which it falls (UCSB, 2017).

2.1.1 Tropospheric Delay

The troposphere is a refractive, non-dispersive, electrically neutrally charged medium that consists of dry gases and water vapor. Electromagnetic signals such as GNSS signals, that pass through the troposphere are deflected and have their propagation velocities and paths altered due to the variable refractive nature of the troposphere (with its refractive index greater than 1). This causes an increase in the time of signal reception by ground receivers, and an increased propagation length known as tropospheric delay. Due to the troposphere’s non-dispersive and electrically neutral characteristics, this delay is independent of the frequencies of the GNSS signals. So, unlike the case for GNSS signal delay mitigation through the ionosphere, the combination of signals with different frequencies less than 30 GHz is ineffective in mitigating the delay through the troposphere.
The refractive index of a medium, \( n \), is the ratio of the speed of light in vacuum, \( c \), to its speed in the medium, \( v \):

\[
n = \frac{c}{v}.
\]  

(2.1)

In a vacuum, \( n \) is unity (that is, it is equal to 1), but since the troposphere is not a vacuum, \( n \) is greater than unity and therefore causes an excess path delay and bending of the GNSS signal. The refractive index is more conveniently represented by the refractivity, \( N \), defined as (Ghoddousi-Fard, 2009):

\[
N = 10^6 (n - 1).
\]  

(2.2)

The tropospheric delay is directly related to the refractive index (or the refractivity), as it results from its integration with respect to height. At every point of the troposphere, the refractivity of a parcel of air can be expressed as a function of atmospheric pressure, temperature, and humidity (Mendes, 1999). The total refractivity of a parcel of air is defined as (Ghoddousi-Fard, 2009):

\[
N = k_1 \frac{P_d}{T} Z_d^{-1} + k_2 \frac{e}{T} Z_w^{-1} + k_3 \frac{e}{T^2} Z_w^{-1},
\]  

(2.3)

where \( P_d \) is the partial pressure of dry air (hPa), \( e \) is the partial pressure of water vapor (hPa), \( T \) is absolute temperature (K), \( k_1, k_2, \) and \( k_3 \) are empirically determined constants, and \( Z_d \) and \( Z_w \) are the compressibility factors of dry air and water vapor respectively, modeled as a function of pressure and temperature:

\[
Z_d^{-1} = 1 + P_d \left[ 57.97.10^{-8} \left[ 1 + \frac{0.52}{T} \right] - 9.4611.10^{-4} \cdot \frac{t}{T^2} \right].
\]  

(2.4)
\[ Z_w^{-1} = 1 + 1.650 \frac{e}{T^3} \left[ 1 - 0.01317. t + 1.75. 10^{-4}. t^2 + 1.44. 10^{-6}. t^3 \right], \]  

where \( t \) represents the temperature in units of Celsius. The first term on the right-hand side of Equation (2.3) is independent of the water vapor content of the troposphere and is therefore known as the dry component of the refractivity; the other terms represent the wet component of the refractivity. An alternative formula for total refractivity, as derived by Davis et al. (1985) is as follows:

\[
N = k_1 R_d \rho + k_2' \frac{e}{T} Z_w^{-1} + k_3 \frac{e}{T^2} Z_w^{-1},
\]  

where, \( \rho \) is the total mass density and:

\[
k_2' = k_2 - k_1 \frac{R_d}{R_{wv}}.
\]  

Terms \( R_d \) and \( R_{wv} \) are the specific gas constants for dry air and water vapor respectively. Unlike equation (2.3), the first term in equation (2.6) is no longer a pure dry component as the total mass density contains the contribution of water vapor (the non-dipole moment). Hence the first term in equation (2.6) is referred to as hydrostatic component as opposed to dry. The rest of the terms in equation (2.6) are referred to as the non-hydrostatic component (Ghoddousi-Fard, 2009). The delay of a GNSS signal \( (d_{trop}) \), along its path through the troposphere \( (dH) \), can be calculated through integration using following expression:

\[
d_{trop} = \int_{path} (n - 1) \, dH.
\]  

Substituting \( n \) in (2.8) for \( N \) in (2.2), the delay, in terms of refractivity is:
\[ d_{\text{trop}} = 10^{-6} \int_{\text{path}} N \, dH. \]  

(2.9)

Considering that the total tropospheric delay can be separated into the hydrostatic and non-hydrostatic (wet) delays, (2.9) as a linear combination of the total delay becomes:

\[ d_{\text{trop}} = 10^{-6} \int_{\text{path}} N_h \, dH + 10^{-6} \int_{\text{path}} N_w \, dH. \]  

(2.10)

Generally, the GNSS signals received by ground receivers do not arrive directly overhead (from the zenith of) the receivers: they arrive from the slant directions. As the delays depend on the actual path that the signals travel through the troposphere, they are therefore also functions of the satellite elevation angle. These slant tropospheric delays \( (d_{\text{trop}}^s) \) at various elevation angles \( (\varepsilon) \) can be expressed in terms of the zenith (overhead) delays and mapping functions, making equation (2.10) become:

\[ d_{\text{trop}}^s = m_h(\varepsilon)d_h^z + m_w(\varepsilon)d_w^z, \]  

(2.11)

where \( m_h(\varepsilon) \) is the hydrostatic mapping function, \( m_w(\varepsilon) \), the wet mapping function, \( d_h^z \), the zenith hydrostatic delay (ZHD), and \( d_w^z \), the zenith wet delay (ZWD).

The closer to the horizon the GNSS satellite is, the bigger the portion of the troposphere the signals will have to travel to reach the ground receiver and, consequently, the higher the impact of the tropospheric refractivity on GNSS signals will be.
In GNSS data analysis, the determination of precise station/point coordinates involves the mitigation of several errors, one of which is the tropospheric delay. This mitigation can be done either through prediction or estimation (Nievinski, 2009). This delay, defined as Zenith Tropospheric/Total Delay (ZTD), is determined through all sets of signal slant delays (SD) from each GNSS receiver station to all observed satellites in its horizon, forming an area on the troposphere roughly like a cone (as depicted in Figure 2.3, adapted from Benevides et al., 2013). This is accomplished through the application of specific mapping functions that project the original slant path of the GNSS signals to the zenith, which is dependent on the satellite’s azimuthal position and elevation angle.

The ZTD of GNSS signals consists of the ZHD and ZWD, and the horizontal gradients to account for azimuthal variations of tropospheric refractions. The ZHD contributes about 90% to the ZTD and can be sufficiently modeled given the observed
surface atmospheric pressure (Wang et al., 2017; Tregoning and Herring, 2006). The ZWD contributes about 10% of the total delay and consists of water vapor which is highly variable in space and time, depending on the complex interplay of several phenomena like convection, precipitation and turbulence. This makes the ZWD difficult to model (Shrestha, 2003). The horizontal gradients provide additional information to describe tropospheric asymmetry. This asymmetry is considered when utilizing GNSS signals from satellites at low elevation angles (Meindl et al., 2004). Taking the horizontal gradient components into account, a more practical expression for the ZTD as stated in Ghoddousi-Fard (2009) is given as:

\[
d_{trop} = m_h(\varepsilon)d_h^z + m_w(\varepsilon)d_w^z + m_g(\varepsilon)[G_{ns}^z \cos(\alpha) + G_{ew}^z \sin(\alpha)],
\]

(2.12)

where \(m_g(\varepsilon)\) is the total gradient mapping function, \(G_{ns}^z\) and \(G_{ew}^z\) are the North-South and East-West horizontal gradients respectively, and \(\alpha\) is the azimuthal angle. The horizontal gradient parameters are needed to correct for the effect of the atmospheric bulge and effects due to changing weather conditions.

The tropospheric delay is an estimated parameter that is mitigated for accurate and reliable point positional information. However, information about the water vapor content of the troposphere can be obtained from the ZWD by converting it to integrated water vapor (IWV) using surface meteorological data (temperature). Due to its high variability in time and space, sudden changes in atmospheric water vapor can result in changes in the local weather. Hence, ZTD and IWV information can then be assimilated into NWM to improve its short-term weather forecasting. GNSS is already being used
for this purpose by several organizations and agencies around the world (de Haan, 2013; Lindskog1, 2017). By analyzing long-term historical GNSS observations and obtaining the ZTD or IWV trends, GNSS can serve as a tool for climate change monitoring (Baltink et al., 2002; Ding et al., 2017).

2.1.2 Tropospheric Delay Models

Tropospheric delays may be derived from actual measurement, from NWM, from models that utilize measured meteorological data (pressure, temperature and humidity), and from empirical, global, and sometimes called ‘blind’ models (models that are built on past meteorological data). Measurement of ZTD can be done using radiosondes, radiometers, light detection and ranging technique or GNSS (Kalita and Rzepecka, 2017).

In GNSS analysis, accounting for the tropospheric delay is vital and can be done by the parameterization and estimation of the delays in the analysis (Nilsson et al., 2013) or by using tropospheric delay models. Tropospheric delay modelling generally involves two parts: use of an a priori model for the tropospheric path delay at the zenith (both for the hydrostatic and wet), which corresponds to the delay that a signal from a satellite at the zenith would have, and use of a mapping function, which accounts for the satellite’s actual elevation angle (Royal Observatory, 2012). While the ZHD can be accurately determined from surface meteorological measurements for use in the analysis of GNSS data, the same cannot be done for the ZWD and so, this delay is usually estimated in the analysis.
2.1.3 Mapping Functions

The mapping function, $m(\varepsilon)$, is defined as the ratio of the delay through the atmosphere at geometric elevation $\varepsilon$, to the delay in the zenith direction. The tropospheric delay is the shortest in the zenith direction and becomes larger with decreasing elevation angle. Several mapping functions have been developed in the past years. The simplest mapping function, which is given by $1/\sin(\varepsilon)$ (Niell, 2000), assumes that spherical constant height surfaces could be approximated as planar surfaces. This is an accurate approximation only for high elevation angles and with a small degree of bending. More complex mapping functions have been developed since most precise analyses require a better formulation (Shrestha, 2003). Currently, the continued fractions in terms of $\sin(\varepsilon)$ are most often used (Andrei, 2007), with different mapping functions for the hydrostatic and wet delays, and horizontal gradients. The general form of the hydrostatic and wet mapping functions is (Herring, 1992):

$$m_{h,w}(\varepsilon) = \frac{1 + \frac{a}{\sin(\varepsilon)+ \frac{b}{\sin(\varepsilon)+c}}}{\sin(\varepsilon)+ \frac{b}{\sin(\varepsilon)+c}}$$

(2.13)

where $m_{h,w}$ is the mapping function, with the $h$ and $w$ subscript indicating it to be hydrostatic or wet, respectively, $\varepsilon$ is the elevation angle, $a$, $b$ and $c$ are empirical coefficients with different values in various mapping functions and are used for the hydrostatic and wet components, respectively as $(a_h,b_h,c_h)$ and $(a_w,b_w,c_w)$. The coefficients $a$, $b$, $c$ are constants or linear functions which depend on surface pressure, temperature, lapse rates, and height.
2.1.4 The Saastamoinen Model

Saastamoinen (1972) assumes that the atmosphere is in hydrostatic equilibrium which follows from the ideal gas law and treats acceleration due to gravity as a function of height. Under hydrostatic equilibrium, the local pressure, which is assumed to be isotropic, provides the balancing force against the atmospheric weight per unit area. The equation of the state of the hydrostatic equilibrium can be written as follows (Ahn, 2016):

\[ dp = -g \cdot \rho \cdot dH, \]  
(2.14)

where \( dp \) is the differential change in pressure (mbar), \( g \) is the acceleration due to gravity at height \( H \) (m/s\(^2\)), \( \rho \) is the total density of air (kg/m\(^3\)), \( dH \) is the differential change in height (m). Rewriting equation (2.14) to introduce the mean gravity acceleration at the centre of mass of the vertical column of the atmosphere above the site \( (g_m) \), we have:

\[ \rho = \frac{1}{g_m} \frac{dp}{dH}. \]  
(2.15)

The hydrostatic component of refractivity in equation (2.6) can then be rewritten as:

\[ N_d = k_i R_d \rho = -k_i R_d \frac{1}{g_m} \frac{dp}{dH}. \]  
(2.16)

Substituting (2.16) into the first part of equation (2.10), we have:

\[ \text{ZHD} = 10^{-6} \int_{H}^{\infty} N_h \, dH = -10^{-6} k_i R_d \frac{1}{g_m} \int_{P_s}^{\infty} dP = 10^{-6} k_i R_d \frac{P_s}{g_m}. \]  
(2.17)
Approximating $g_m$ by the Saastamoinen equation as given by Davis et al. (1985), we have:

$$g_m = 9.784 f(\varphi, H),$$

(2.18)

$$f(\varphi, H) = 1 - 0.00266 \cos 2\varphi - 0.00000028 H,$$

(2.19)

$$ZHD = \frac{0.0022768 P_s}{f(\varphi, H)},$$

(2.20)

where $P_s$ is the pressure at the earth surface (in millibars); $\varphi$ is geodetic latitude at the station (in radians) and $H$ is the geodetic height at the station (in metres). For the ZWD, Saastamoinen (1972) proposes the calculation based on ideal gas laws using a simple relation:

$$ZWD = \frac{0.0022768 P_s}{f(\varphi, H)} (1255 + 0.05T_s) \frac{e_s}{T_s},$$

(2.21)

where $T_s$ is the temperature at the earth surface and $e_s$ is the water vapor pressure at the earth surface (Liu et al., 2017).

**2.1.5 The Niell Model**

The Neil Model is a combination of the Saastamoinen delay model, as described above, together with Neil mapping functions (Niell, 1996), as described later in section 2.1.7 of this chapter. The coefficients $a, b$ and $c$ used in the dry and wet components of the models are calculated based on the interpolation of the average and seasonal variation (amplitude) values as functions of latitude and time (Dodo and Idowu, 2010).
2.1.6 Numerical Weather Model

The Numerical Weather Models (NWM) are three-dimensional models of the atmospheric conditions in the troposphere. The models contain predicted information about different meteorological parameters such as temperature, relative humidity, geopotential height, and two horizontal wind components. The purpose of the NWM is to predict the future state of the atmospheric circulation from the information on the present conditions by using numerical approximations of the dynamical equations describing the atmospheric circulation (Andrei, 2007). They use a series of mathematical models to generate either short-term weather forecasts or longer-term climate predictions. Various observations collected from a dense meteorological sensor network, including ground meteorological stations, radiosondes, weather balloons, remote sensing weather satellites and commercial aircraft are assimilated into the NWM. Benefiting from these weather sensors and increased power of the supercomputers, NWM can provide the best achievable analysis or prediction on the actual, physical conditions of the atmosphere, which contain the necessary information for GNSS tropospheric delay estimation (Yang et al., 2013).

Numerical weather prediction depends on a set of non-linear partial differential equations that describe the physical laws influencing the change of atmospheric conditions such as pressure, temperature, humidity and wind. It also depends on the numerical solutions regarding these equations (Gutman and Benjamin, 2001). The realization of NWM is accomplished after solving the initial value problem, that is, the initial conditions or state of the atmosphere. This process is referred as data
assimilation, while the resolved atmospheric state is named as analysis. Information about tropospheric delays and horizontal gradients, as well as the precipitable or integrated water vapor at a given location can also be supplied by NWM (Hobiger et al., 2008; Zus et al., 2014).

Over the years, NWM have proven to be highly useful for the determination of the mapping functions describing the growth of the tropospheric path delay with increasing zenith distance. By raytracing through meteorological fields, the determined path delays were only used to obtain parameters of the mapping functions and not considered in space geodetic data processing. However, NWM have been improved regarding their spatial and temporal resolution. Further, an increasingly large number of small-scale phenomena are now being considered in the model runs. The enhanced accuracy and precision have made it feasible to utilize ray-traced tropospheric delays directly for the analysis of space geodetic applications (Hobiger et al., 2008).

NWM can be divided into two categories regarding their size of coverage and horizontal resolution. The first are the regional NWM which are limited in coverage to countries operating them, or its surrounding regions. They often have a horizontal resolution of 2-10 km, and examples are the Japanese Meteorological Agency meso-scale weather model, the High-Resolution regional model and the COSMO-DE model of the Deutscher Wetterdienst, and the ALADIN-Climate/CZ of Czech Hydrometeorological Institute. A variety of other regional NWM are also available from many countries like the United Kingdom, Australia, United States and Canada. The second are the global NWM which show a resolution of 10-15 km and have much wider
coverage than the regional NWM. Some of the most widely used global NWM are the European Center for Medium-range Weather Forecasts (ECMWF), the National Centers for Environmental Prediction (NCEP) provided by the National Oceanic and Atmospheric Administration of the United States, and the Canadian Meteorological Centre’s Global Environment Mesoscale. The regional NWM are usually applied for short-term forecasts for specific regions, ranging from one to three days. However, it is noteworthy that the regional models require accurate information about the boundary conditions, which are mainly offered by the global NWM. On the other hand, the global NWM fit better for the work related to the medium-range forecast (longer than two days) and climate studies (Kalnay, 2003).

For this thesis research, we used the global models of the ECMWF 40-year reanalysis (ERA-40) (Simmons and Gibson, 2007) and NCEP Department of Energy Atmospheric Model Inter-comparison Project reanalysis 2 (NCEP II) (Kanamitsu et al., 2002).

2.1.7 Neill Mapping Function (NMF)

The Neill mapping function is an empirical mapping function that was developed using radiosonde data from profiles of United State’s Standard Atmosphere for the northern hemisphere, stored for the months January and July, to the elevation of 3°, over a wide range of latitudes. It is based on the continued fraction form (Nieell, 1996) as equation (2.13), with three coefficients normalized to unity at the zenith (Herring, 1992),
to calculate the coefficients of $a$, $b$, $c$ for latitude by linear interpolation for both the hydrostatic part and non-hydrostatic part.

The NMF uses the day of the year (DOY) and station latitude as input and is symmetric around the equator, inverting the seasons the southern hemisphere by adding half a year to the phase of the southern latitudes. The derivation of the $a$ and $b$ coefficients of the hydrostatic mapping function depend on the DOY, latitude and height above sea level of the observing site, while the dependence of the wet mapping function is only on the site latitude. Thus, no meteorological data is needed, but a height correction term is required and is given in equation (2.23). For the hydrostatic mapping function, the coefficients are calculated as:

$$a = a_0 + A \cos\left(\frac{\text{doy} - 28}{365.25} \cdot 2\pi\right),$$  \hspace{1cm} (2.22)$$

where the $a_0$ mapping function coefficient is the average value and $A$, the amplitude, both as functions of latitude. For the wet mapping function, only an interpolation in latitude for each parameter is needed. The form adopted for the analytic height correction is:

$$\Delta m(\varepsilon) = \frac{dm(\varepsilon)}{dh} H,$$  \hspace{1cm} (2.23)$$

where $$\frac{dm(\varepsilon)}{dh} = \frac{1}{\sin(\varepsilon)} - f(\varepsilon, a_{ht}, b_{ht}, c_{ht}).$$  \hspace{1cm} (2.24)$$

$H$ is the height of the site above sea level and $f(\varepsilon, a_{ht}, b_{ht}, c_{ht})$ is the three-term continued fraction (equation (2.13)), with $a_{ht}, b_{ht}$ and $c_{ht}$ determined by least squares fit to the height corrections at nine elevations from $3^\circ$ to $90^\circ$. 
2.1.8 Vienna Mapping Function 1 (VMF1)

The Vienna Mapping Functions 1 (Böhm et al. 2006b) is derived from the European Center for Medium-range Weather Forecasting (ECMWF) numerical weather model (NWM), with the underlying functional formulation based on the continued fraction form of Marini (1972), with three coefficients $a$, $b$, and $c$, normalized to yield unity at the zenith (Herring, 1992). The $a$ coefficient is determined by one-dimensional ray-tracing in the zenith direction and at an initial elevation angle of 3.3 degrees with data from the NWM, that is, no asymmetries around the sites are considered as only refractivity at the site vertical is used for this type of ray-tracing. The $b$ and $c$ coefficients are determined empirically and represented by analytical functions of site latitude and the DOY. The $a$ coefficient is determined on a 6-hourly basis, which allows the mapping functions to capture the small-scale temporal variations in the slant delay better than classical mapping functions which rely only on climatology. The coefficients are provided for all Very Long Baseline Interferometry and selected GNSS sites and are also provided in a gridded format to make it available at any location. For the gridded format, the $a$ coefficients are determined on a $2.5^\circ \times 2.5^\circ$ horizontal grid and reduced to a zero altitude.

2.1.9 Global Mapping Function (GMF)

The Global Mapping Function -GMF- (Böhm et al., 2006a) is derived using $15^\circ \times 15^\circ$ global grids of monthly mean profiles for pressure, temperature, and humidity from the ERA-40 data. The hydrostatic and wet $a$ coefficients, $ah$ and $aw$, are determined for
the period September 1999 to August 2002 applying the same strategy used for VMF1. While the $b$ and $c$ coefficients are determined by empirical equations (from VMF1), the $a$ coefficients are derived by a single ray-trace at an initial elevation angle of $3.3^\circ$, thus obtaining 36 monthly values for the hydrostatic and wet $a$ parameters of 312 grid points. The hydrostatic coefficients are reduced to mean sea level by applying the height correction given in equation (2.22).

The mean values, $a_0$, and the annual amplitudes, $A$, of a sinusoidal function are fitted to the time series of the $a$ parameters at each grid point, with the phases referred to January 28, corresponding to the NMF. The standard deviations of the monthly values at the single grid points with respect to equation (2.22) increase toward higher latitude from the equator, with a maximum value of 8 mm (equivalent station height error) in Siberia. For the wet component, the standard deviations are smaller with maximum values of about 3 mm at the equator (Böhm et al., 2006).

$$a_0 = \sum_{n=0}^{9} \sum_{m=0}^{n} P_{nm} (\sin \varphi). [A_{nm} \cdot \cos m. \lambda + B_{nm} \cdot \sin m. \lambda]. \quad (2.25)$$

Then, the global grid of the mean values, $a_0$, and of the amplitudes, $A$, for both the hydrostatic and wet coefficients of the continued fraction form are expanded into spatial spherical harmonic coefficients up to degree and order 9 (according to equation (2.25) for $a_0$) in a least-squares adjustment. The residuals of the global grids of $a_0$ and $A$ values to the spherical harmonics are in the sub-millimeter range (in terms of station height). The hydrostatic and wet coefficients $a$ for any site coordinates and day of year can then be determined using equation (2.22) (Böhm et al., 2006).
2.2 Nigerian Climate

As shown in Figure 2.4 (Blogger.com, 2017), Nigeria is a tropical country in West Africa bounded in the north by the Niger Republic and Chad, in the west by Benin Republic, Cameroon in the east, and the Atlantic Ocean in the south. The country is covered by approximately 13,000 km² of water (1.4%) and 98.6% of land cover ranging from thick mangrove forests and dense rain forests in the south to a near-desert condition in the northeastern corner of the country (Ibe and Nymphas, 2010).

![Map of Nigeria in Africa](image-url)

Figure 2.4 – Map of Nigeria in Africa
The Nigerian climate is dominated by the influence of three main wind currents known as the trade winds. They are the Tropical Maritime (TM) air mass, the Tropical Continental (TC) air mass and the Equatorial Easterlies (EE). TM originates from the southern high-pressure belt located off the Namibian coast and is wet as it collects moisture along its way over the Atlantic Ocean. The TC has the high-pressure belt north of the Tropic of Cancer as its origin. This air mass is always dry because of the little moisture it collects along its way. TM and TC meet along a starting surface called the Inter-Tropical Discontinuity (ITD). The EE air mass is an erratic cool air mass that comes from the east and flows in the upper atmosphere along ITD. This air mass penetrates occasionally to actively undercut the TM or TC and gives rise to squall lines or dust devils (Oguntunde et al., 2011; Oginni and Adebamowo, 2013).

The Atlantic Ocean and Cameroon Mountains, which form the southern and eastern boundaries, and the Sahara Desert, above the north of Nigeria, exert influences on the country's climate systems. The two main seasons that characterize Nigeria are the dry and rainy (wet) seasons. The dry season is accompanied by a dust-laden wind from the Sahara Desert, known as Harmattan, which is brought by the TC, while the TM from the Atlantic Ocean heavily influences the rainy season. The local climate in the north central region may also be more influenced by the high elevation in the region than the south (Eludoyn et al., 2014). Rainfall/precipitation is the key climatic variable used to describe the Nigerian climate and seasons, with amount and seasonal distribution being the two key aspects of precipitation variation. Generally, the amount of total annual precipitation increases as one moves from north to south. In addition, rainfall becomes more
seasonally concentrated moving northward. Most of Nigeria is characterized by a monsoonal rainfall regime in which much of the precipitation falls within a well-defined period. There is also a marked alternation of wet and dry seasons in most areas. In the southern coastal regions, rainfall commences in approximately March/April, spreads through the mid region in May/June, and reaches the north in June/July, reaching its peak over the middle and northern regions between July and September. In northern Nigeria, the dry season is typically six to eight months. In southern Nigeria, the dry season is much shorter, lasting only about two to four months.

According to Olaniran (1987), rainfall regimes in the tropics are controlled by two groups of factors, the first of which is active over large areas while the other is only effective over much smaller regions. The first group of factors consists of elements of the general circulation of the tropical atmosphere: the ITD and the sub-tropical high-pressure cells. The second group of factors is made up of local factors such as convection, topography, local circulation types, all of which are effective only over small areas. Based on the effects of the first group of factors a zonal pattern of rainfall regime can be developed for the tropics. Over many tropical areas, the areal distribution depicts the double maximum rainfall regime. Nigeria for example shows this pattern from the coast northward on the western side of the country but a single maximum rainfall regime for the eastern section of the country. Also, the intensity and duration of the shorter period of less rain in southern Nigeria decreases progressively eastward such that the double peak rainfall is not recognizable beyond 5°E. Similar distortions to the zonal pattern of seasonal variation types of rainfall for places outside West Africa have been noted. The
irregularities have been explained in terms of the influence of convection, topography and local circulation types amongst others. These processes are effective over much smaller regions and constitute the second group of factors affecting rainfall regimes (Olaniran, 1987).

Studies carried out by Willoughby et al. (2002) on the seasonal variations of radio refractivity gradients in Nigeria also confirms the double peak of rainfall in Nigeria. A slight depression in mean gradients during the wet months was observed in August. This diminished mean gradient also reflected in the wet term gradient and its standard deviation, coinciding with the brief reduction of rainfall along the coast in August. During this August drought, water vapor density at the surface is low, leading to a drop in the refractivity gradient. Despite the great depth and humidity of the maritime air existing near the coast, the duration of this drought was found to be about three weeks and occurs only in the southern part of Nigeria. Stations that experienced this phenomenon are in regions that exhibit a double rainfall maximum with peaks at about May/June and September/October.

Another climatic variable used to describe the Nigerian climate is relative humidity. In general, relative humidity is higher in the southern part of the country. During the wet season, relative humidity is typically above 80% in southern Nigeria. Relative humidity in the northern half of the country is lower on average in the wet season, typically in the range of 60-80%. During the dry season, northern Nigeria seldom sees relative humidity above 40% (Hoepner, n.d.).
Generally, the Nigerian climate is characterized by three strong latitudinal zones covering the southern, middle and northern areas of the country: The Tropical Rainforest Climate, the Tropical Savanna Climate, and the Highland or Montane Climate, as depicted in Figure 2.5 (Eludoyin, 2015). The Tropical Rainforest climate characterizes the southern region. It is sub-grouped into the Tropical Wet and the Tropical Wet and Dry climates. It is characterized by short dry seasons, small temperature range throughout the year, and usually convectional storms/heavy rainfall because of its proximity to the equator.

The Tropical Savanna climate comprises the Guinea, Sudan and Sahel Savanna, and characterizes most of the central and northern regions. The Guinea belt occupies the limits of Tropical Rainforest climate and extends to the central part while the northern fringe is occupied by the Tropical (Sudan) Savanna climate. The north-eastern fringes exhibit the Sahelian climate. Overall, the Tropical Savanna climate exhibits a well-marked single peak rainy season, with less annual rainfall than the Tropical Rainforest climate, and a long dry season. Compared to the south and central parts of Nigeria, the Sahel climate has even less Rainfall, with short rainy season and temperatures that go as high as 44°C (Fontaine, 2013). The dry season is hot and dry with the harmattan wind prevailing throughout this period.

The Highland or Montane climate is experienced on highlands regions in Nigeria. Highlands with Montane climate in Nigeria exceed 1520 m above sea level. Because of their location in the tropics, this elevation provides the settlements on the mountains and
the plateau regions standing above this height, a cool Montane climate (Eludoyin et al., 2014).

![Figure 2.5 – Classification of the Nigerian climate]

2.3 Time Series Analysis

A time series is a sequence of measurements of the same variable, often at regular time intervals, over time. Examples of time series are heights of ocean tides, length of day, counts of sunspots, etc. Time series are used in statistics, signal processing, pattern recognition, econometrics, mathematical finance, weather forecasting, earthquake prediction, electroencephalography, control engineering, astronomy, communications
engineering, and largely in any domain of applied science and engineering which involves temporal measurements.

Time series analysis comprises methods for analyzing time series data to extract meaningful statistics and other characteristics of the data. Time series are serially correlated in contrast to basic data analysis where the assumption of identically and independently distributed data is key. The purpose of time series analysis is to visualize and understand dependencies in past data, and to exploit them for forecasting future values or occurrences. While some simple descriptive techniques do often considerably enhance the understanding of the data, a full analysis usually involves modeling the mechanism that is assumed to be the generator of the observed time series. Once a good model is found and fitted to the data, the analyst can use that model to forecast future values and produce prediction intervals, or he can generate simulations, for example to guide planning decisions. The dominant main features of many time series are trend and seasonal variation. These can either be modeled deterministically by mathematical functions of time or are estimated using non-parametric smoothing approaches. Yet another key feature of most time series is that adjacent observations tend to be correlated, i.e. serially dependent. Much of the methodology in time series analysis is aimed at explaining this correlation using appropriate statistical models (Dettling, 2016).

Due to the nature of measurements, the observed time series are assumed to be composed of two main constituents: signal and noise. The noise, which is the disturbing part on observations, can be random and/or systematic. In the concept of noise, the uncorrelated random noise with constant spectral density is assumed as ideal. This kind is
also called as white noise. However, in practice, the observables mostly include non-white random noise, which is a band–limited random function of time. Systematic noise may also have systematic and non-systematic parts and can be modeled with certain mathematical forms. Non-systematic noise can include datum shift, trend etc. and renders the series non-stationary. While analyzing the time series, generally the possible trend, which may fit linear, quadratic, exponential and so on expressions, is identified and removed in the first step to avoid non-systematic noise (Erol, 2010; Erol et al., 2006).

GNSS-derived ZTDs have been highly useful in the field of meteorology and climatology on diverse time scales. Studies of trends and seasonal oscillations/variations have used data from GNSS to analyse the seasonality of ZTD through time series analysis, reporting the presence of seasonal ZTD cycles (Baldysz et al., 2015; Baldysz et al., 2016). Jin et al. (2007) investigated the seasonal variability of ZTD and its implications on climate. In their work, the ZTD time series of 150 globally distributed IGS stations were studied to determine their secular trends and seasonal variations. Their results indicate significant annual ZTD variations at all IGS stations with larger amplitudes for stations near oceanic coasts than those in the continental inland. Also, larger amplitudes of annual ZTD variation are mostly found at middle latitudes and smaller amplitudes at higher latitudes and equatorial areas. Phases of these annual ZTD variations in the Southern Hemisphere occur around February, which is its summer time, and around August, which coincides with the summer time in the northern hemisphere.

In Nigeria, a study to comprehend the spatial and temporal variability of the GNSS-derived ZTDs of NIGNET stations during the period 2010–2014 was conducted
by Isioye et al. (2017). In their findings, diurnal ZTD cycles exhibit seasonal dependence, with larger amplitudes in the rainy (wet) season and smaller ones in the harmattan (dry) season. Notably, the ZTD of the northern stations in the country reached high amplitudes in the months of June, July and August, which are the peak of the wet season, characterized by very high rainfall.
3. Tropospheric Delay Estimation Techniques and Least Squares Spectral Analysis

As mentioned in the previous chapter, ZTD estimates may be derived from a variety of models or from measurements from various instruments. This chapter will address the two techniques/sources used to obtain zenith tropospheric delays used in this thesis research. They are the processing of GNSS observations through Precise Point Positioning and ray-tracing using meteorological data from numerical weather models. The chapter also discusses the Least Squares Spectral Analysis used for analyzing the ZTD time series.

3.1 Estimation of Tropospheric Delays through Precise Point Positioning

One of the common strategies in use today for the processing of GNSS observations is Precise Point Positioning (PPP). PPP is a positioning technique that uses single-frequency or un-differenced dual-frequency pseudo range and carrier phase observations of a single receiver (to remove the first order effect of the ionosphere), along with the precise satellite orbit and clock products to determine cm-level positioning accuracy (Zumberge et al., 1997). The precise products are produced by various GNSS scientific and research organizations by means of post-processing GNSS data from global networks of reference stations. These products provide information of better quality than the broadcast ephemeris message. The PPP approach arose from the
The advent of widely available precise GPS satellite orbit and clock data products with centimeter accuracy.

The advantage of this technique is that no regional network correlations will be introduced, and no reference station data is explicitly required for data processing. This allows the checking of the consistency of the introduced orbit, clock and atmosphere error models. Omitting the building of baselines significantly reduces the required processing time compared to the network (baseline) solution. On the other hand, the remaining hardware biases, both from satellite and receiver, must be accounted for carefully, with the necessary error models made available by global or regional services.

### 3.1.1 The PPP Observation Model

The PPP algorithm typically uses code and carrier-phase observations from a dual-frequency receiver, and precise satellite orbits and clocks information as input to calculate precise receiver coordinates, clocks errors, estimates of the zenith wet delays, and phase ambiguities. In PPP, the carrier-phase measurements are treated as independent measurements, rather than being used to filter the pseudo ranges as is done in other positioning techniques, which leads to an ambiguity parameter estimation for each satellite (Leandro, 2009; Karabatić, 2011; Kouba et al., 2017).

The carrier-phase and code observation equations at a given epoch, and for a given satellite, are:
\[
\Phi_i = \rho + c(dT_r - dt) - I_i + T + \lambda_i N_i + \Delta_{rel} + \lambda_i \alpha + \lambda_i \alpha_r + \lambda_i \alpha_s + \Delta_{pcv,i} + m_{\Phi,i} + \varepsilon_{\Phi,i},
\]

(3.1)

\[
P_i = \rho + c(dT_r - dt) + I_i + T + \Delta_{rel} + c\beta_r + c\beta_s + m_{P,i} + \varepsilon_{P,i},
\]

(3.2)

where \( i \) is the carrier frequency (L1 or L2), \( \Phi_i \) and \( P_i \) are carrier-phase and code measurements in metres, on the \( i \) frequency respectively, \( c \) is the speed of light in meters per second, \( dT_r \) and \( dt \) are the receiver and satellite clock errors respectively in seconds, \( T \) is the tropospheric/neutral atmospheric slant delay in meters, \( I_i \) is the \( i \) frequency ionospheric slant delay in meters, \( \Delta_{rel} \) is the range correction due to relativity effects in metres, \( \Delta_{pcv,i} \) is the phase centre variation delay of the \( i \) frequency in metres, \( \lambda_i \) is the carrier-phase wavelength on the \( i \) frequency in meters, \( N_i \) is the carrier-phase integer ambiguity on the \( i \) frequency in cycles, \( \alpha_r \) and \( \alpha_s \) are the receiver and satellite phase biases respectively, \( \beta_r \) and \( \beta_s \) are the receiver and satellite code biases respectively, \( m_{\Phi,i} \) and \( m_{P,i} \) are the code and carrier-phase multipath errors respectively, on the \( i \) frequency in meters, \( \varepsilon_{\Phi,i} \) and \( \varepsilon_{P,i} \) are the remaining un-modeled errors and white noise for the carrier-phase and code measurements on the \( i \) frequency respectively, in meters, and \( \rho \) is the geometric distance between the satellite and receiver antennas, in meters, computed as a function of the satellite coordinates \((X_s, Y_s, Z_s)\) and receiver coordinates \((X_r, Y_r, Z_r)\) as:

\[
\rho = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}.
\]

(3.3)

Elimination of the first-order ionospheric delay from carrier-phase and code measurements, involves the use of an ionospheric-free combination of the two
frequencies, L1 and L2. Due to the dispersive nature of the ionosphere, the magnitude of the first-order delay is inversely proportional to the frequency squared, as:

\[ I_i = \frac{40.3 \, TEC}{f_i^2}, \]  

(3.4)

where \( I_i \) is the ionospheric delay on the \( i \) frequency in metres, \( f_i \) is the signal frequency and \( TEC \) is the Total Electron Content.

For PPP, the satellite's position is held fixed or tightly constrained, and the satellite clock corrections are also considered as known. Furthermore, the tropospheric delay is expressed as a product of the zenith delay \( (T_z) \) and mapping function \( (M) \), which relates slant path delay \( (T) \) to zenith delay. To achieve high accuracy with PPP, all necessary observation corrections such as tides, relativistic effects, and receiver and satellite antenna phase center variation among others, are considered and applied (Leandro, 2009). Satellite antenna offsets are obtained from the official IGS antenna calibration file. The precise modeling of Earth dynamics (causing variations of the static receiver coordinates with respect to the terrestrial reference frame) is normally based on the International Earth Rotation and Reference Systems Service (IERS) recommendations (IERS Conventions, 2010). Such models include solid Earth tides, ocean loading and Earth Rotation. Detailed explanations of these models and other PPP correction models are given in Karabatić (2011) and Kouba et al. (2017).

Applying the corrections and the models for relativistic effects, phase center variations, phase wind-up effect and system biases, the carrier and code ionospheric–free linear combinations can be formed as follows:
\[ \Phi_{if} = \rho + c(dT_r - dt) + MT_z + \lambda_{if}N_{if} + \varepsilon_{\Phi,if}, \]  
(3.5)

\[ P_{if} = \rho + c(dT_r - dt) + MT_z + \varepsilon_{P,if}, \]  
(3.6)

where:

\[ \Phi_{if} = \frac{f_1^2}{f_1^2 - f_2^2} \Phi_1 - \frac{f_2^2}{f_1^2 - f_2^2} \Phi_2, \]  
(3.7)

\[ P_{if} = \frac{f_1^2}{f_1^2 - f_2^2} P_1 - \frac{f_2^2}{f_1^2 - f_2^2} P_2, \]  
(3.8)

\[ \lambda_{if} = \frac{f_1^2}{f_1^2 - f_2^2} \lambda_1 - \frac{f_2^2}{f_1^2 - f_2^2} \lambda_2, \]  
(3.9)

and

\[ N_{if} = \frac{f_1^2}{f_1^2 - f_2^2} N_1 - \frac{f_2^2}{f_1^2 - f_2^2} N_2. \]  
(3.10)

The terms \( \Phi_{if}, P_{if}, \lambda_{if}, \) and \( N_{if} \) are the ionospheric-free carrier-phase measurement, code measurement, carrier-phase wavelength and carrier-phase integer ambiguity respectively; \( \varepsilon_{\Phi,if} \) and \( \varepsilon_{P,if} \) are the ionospheric-free carrier-phase and code observation noise respectively, including the multipath errors; \( f_1 \) and \( f_2 \) are the L1 and L2 carrier frequencies respectively; \( \Phi_1 \) and \( \Phi_2 \) are the phase measurements on L1 and L2 carrier frequencies respectively; \( P_1 \) and \( P_2 \) are code measurements on L1 and L2 frequencies respectively; \( \lambda_1 \) and \( \lambda_2 \) are wavelengths of the L1 and L2 carrier frequencies respectively; and \( N_1 \) and \( N_2 \) are the L1 and L2 carrier-phase integer ambiguities respectively. A drawback of the ionospheric-free combination is the three-fold increase in magnitude of the measurement noise with respect to L1 or L2 observations.
The observations coming from all the satellites are processed together in a least squares adjustment that solves for the different unknowns, namely the receiver coordinates, the receiver clock, the zenith wet delay and the phase ambiguities. The design matrix needed for the adjustment follows from the linearization of the observation equations around approximate parameter values that consist of the partial derivatives of equations (3.5) and (3.6) with respect to the receiver coordinates, the receiver clock, the zenith wet delay and the phase ambiguities. Such adjustment can be done in a single step called batch adjustment (with iterations) or within sequential adjustment or filter (with or without iterations) (Kouba et al., 2017).

Ignoring the noise terms, since they are the non-modeled part of the positioning model, and assuming a-priori values for all parameters, the observation equations for phase and code measurements, as given in Leandro (2009), can be written as follows:

\[ \Phi_{if} = (\rho_0 + \delta\rho) + c((dT_{r0} + \delta dT_r) - dt) + M(T_{z0} + \delta T_z) + \lambda_i\delta_{Nf,0} + \delta N_{f,0}, \]  
\[ \text{ (3.11) } \]

and

\[ P_{if} = (\rho_0 + \delta\rho) + c((dT_{r0} + \delta dT_r) - dt) + M(T_{z0} + \delta T_z). \]  
\[ \text{ (3.12) } \]

The terms with the zero subscript represent the a-priori values, and the errors from using the a-priori values are represented with the delta terms, in other words, the difference between the observed value and the a-priori value of each parameter. These errors are the updates for computation, so the a-priori values can be corrected to approach the observed values of each parameter. Rewriting the equations (3.11) and
Using partial derivatives of each parameter with respect to the observation, we have:

\[
\Phi_{if} = \rho_0 + \frac{\delta \Phi_{if}}{\delta X_r} \delta X_r + \frac{\delta \Phi_{if}}{\delta Y_r} \delta Y_r + \frac{\delta \Phi_{if}}{\delta Z_r} \delta Z_r + \frac{\delta \Phi_{if}}{\delta T_r} \delta d T_r + c d T_{\tau 0} + \frac{\delta \Phi_{if}}{\delta T_z} \delta T_z + \lambda_{if} N_{if,0} + \frac{\delta \Phi_{if}}{\delta N_{if}} \delta N_{if,0},
\]

(3.13)

and

\[
P_{if} = \rho_0 + \frac{\delta P_{if}}{\delta X_r} \delta X_r + \frac{\delta P_{if}}{\delta Y_r} \delta Y_r + \frac{\delta P_{if}}{\delta Z_r} \delta Z_r + \frac{\delta P_{if}}{\delta T_r} \delta d T_r + c d T_{\tau 0} + \frac{\delta P_{if}}{\delta T_z} \delta T_z - c d t + M T_{\tau 0} + \frac{\delta P_{if}}{\delta T_z} \delta T_z,
\]

(3.14)

where \(X_r, Y_r\) and \(Z_r\) are the receiver cartesian coordinates, \(T\) is the zenith neutral atmosphere delay and \(N\) is the ambiguity. Rearranging equations (3.13) and (3.14), with the partial derivatives as the subject of the formula, we have:

\[
\frac{\delta \Phi_{if}}{\delta X_r} \delta X_r + \frac{\delta \Phi_{if}}{\delta Y_r} \delta Y_r + \frac{\delta \Phi_{if}}{\delta Z_r} \delta Z_r + \frac{\delta \Phi_{if}}{\delta T_r} \delta d T_r + \frac{\delta \Phi_{if}}{\delta T_z} \delta T_z + \frac{\delta \Phi_{if}}{\delta N_{if}} \delta N_{if,0} = \Phi_{if} - \rho_0 - c d T_{\tau 0} + c d t - M T_{\tau 0} - \lambda_{if} N_{if,0},
\]

(3.15)

and

\[
\frac{\delta P_{if}}{\delta X_r} \delta X_r + \frac{\delta P_{if}}{\delta Y_r} \delta Y_r + \frac{\delta P_{if}}{\delta Z_r} \delta Z_r + \frac{\delta P_{if}}{\delta T_r} \delta d T_r + \frac{\delta P_{if}}{\delta T_z} \delta T_z = P_{if} - \rho_0 - c d T_{\tau 0} + c d t - M T_{\tau 0}.
\]

(3.16)
The partial derivatives are evaluated as:

\[
\frac{\delta \phi_{if}}{\delta X_r} = \frac{\delta P_{if}}{\delta X_r} = \frac{X_s - X_r}{\rho_0},
\]

\[
\frac{\delta \phi_{if}}{\delta Y_r} = \frac{\delta P_{if}}{\delta Y_r} = \frac{Y_s - Y_r}{\rho_0},
\]

\[
\frac{\delta \phi_{if}}{\delta Z_r} = \frac{\delta P_{if}}{\delta Z_r} = \frac{Z_s - Z_r}{\rho_0},
\]

where \( X_s, Y_s \) and \( Z_s \) are the satellite cartesian coordinates at the time of signal transmission,

\[
\frac{\delta \phi_{if}}{\delta d_T} = \frac{\delta P_{if}}{\delta d_T} = c,
\]

\[
\frac{\delta \phi_{if}}{\delta \tau_z} = \frac{\delta P_{if}}{\delta \tau_z} = mf_w,
\]

\[
\frac{\delta \phi_{if}}{\delta N_{if}} = \lambda_{ifr},
\]

where \( mf_w \) is the wet mapping function.

### 3.1.1.1 University of New Brunswick’s GNSS Analysis and Positioning Software

The GNSS Analysis and Positioning Software (GAPS) is a PPP software package developed at the UNB, for positioning, data analysis and quality control. GAPS can be used either as a web-based positioning service with which users can upload GNSS observations to be processed or as a command line executable version,
which can be used to process large amounts of GNSS data in a fast and convenient manner. Users of GAPS are provided with accurate satellite positioning from a single GNSS receiver either in static or kinematic mode. Precise orbit and clock products, provided by the International GNSS Service or by Natural Resources Canada, are used to achieve centimeter-level positioning in static mode and decimeter-level positioning in kinematic mode given a sufficient convergence period (Urquhart et al., 2014).

The GAPS PPP engine uses equations (3.5) and (3.6) as its functional model. Observation equations for phase and code measurements are as given in Section 3.1.1. Processing is done on an epoch-by-epoch basis, using a sequential weighted least squares filter to estimate the station position, receiver clock, zenith wet delay and one ambiguity per satellite (treated as a real number). Taking into consideration the asymmetric nature of the troposphere, its modeling improves the repeatability of the PPP time series (Bar-Sever et al. 1998). GAPS uses the linear horizontal gradient formulation of Chen and Herring (1997) to model the asymmetric tropospheric delay, thereby producing estimates (optionally) of two troposphere gradient parameters, $G_{nS}^z$ and $G_{eW}^z$. These are the North-South and East-West horizontal gradients respectively.

In GAPS, the least squares observation equation containing the design matrix, the weight matrix and the misclosure vector with the addition of all necessary correction models are detailed in Leandro (2009). According to Urquhart et al. (2014), the parameter updates are computed at every epoch of observation as:

$$\delta = (A^t P A + C_\delta^{-1})^{-1} A^t P w,$$

(3.23)
where $\delta$ is the update vector (vector of unknowns), $A$ is the design matrix, $P$ is the weight matrix, $C_\delta$ is the covariance matrix of the parameters, $w$ is the misclosure vector.

The covariance matrix is also updated at each epoch according to:

$$
C_{\delta,t} = (A^tPA + C_{\delta,t-1}^{-1})^{-1} + C_n,
$$

(3.24)

where $C_n$ is the process noise matrix, for which the values vary depending on the type of parameter. The values of $C_n$ are usually constant and $t$ and $t-1$ are epoch indicators for $C_\delta$. Both the parameters’ update vector and its covariance matrix consist of updates, and variance and covariances, respectively, for 3-dimensional (3-D) coordinates, receiver clock, residual neutral atmosphere delay and carrier-phase ambiguities of the observed satellites. The horizontal gradient terms too can be included as additional parameters if the option to compute them is selected in GAPS, as is the case for this thesis research.

Zeros can be used if there are no approximate values for parameters such as receiver clock error and receiver position. In GAPS, the ambiguities are always initialized with zeros, and the tropospheric delay with the estimate provided by a variety of models. The VMF1 model is used by default and was the selected option for this thesis. The details of the full processing strategy and options for GAPS is listed in [http://gaps.gge.unb.ca/strategy.html](http://gaps.gge.unb.ca/strategy.html), but for this thesis, the table of the processing options used is given in Appendix A. Other than the selection of the option to compute tropospheric horizontal gradients, all other default options for processing were used in
this thesis research, which include the use of only observations from the GPS satellite constellation.

3.1.1.2 International GNSS Service (IGS) Zenith Path Delay

The International GNSS Service, formerly International GPS Service was established as a service of the International Association of Geodesy in 1994 to provide GNSS data and products free of charge to all interested users. It is a voluntary, non-commercial organization with more than 200 contributing institutions, more than a dozen regional and operational data centers, four global data centers, eleven (11) analysis centers (ACs) and some associate or regional ACs. The United States Naval Observatory, which is one of the 11 analysis centers, produces IGS Final Troposphere Estimates (FTE) for most of the stations of the IGS network. Each daily file provides five-minute-spaced estimates of total troposphere zenith path delay (ZPD), and north-south and east-west gradient components, along with their respective standard deviations.

ZPD values are directly estimated from raw GPS-only range measurements from the IGS stations, using PPP from Bernese GNSS Software v.5.0, developed at Astronomical Institute, University of Bern, Switzerland (Dach et al., 2007), and using the IGS combined final orbit and clock products. The IGS FTE is not a combination of the independent solutions from the ACs. The reason for this is to avoid inconsistencies from ACs using different estimation methods, from the number of ACs contributing their solutions, or from an inconsistent set of common stations used among the
contributing ACs. Each site–day’s results are available approximately three weeks following the observation day (three weeks after the IGS Final GPS products availability) (Hackman et al., 2015; Byun and Bar-sever, 2009).

A-priori modeling of the tropospheric delay is performed using the GMF and the Niell Model, which is a combination of the NMF and Saastamoinen model (standard meteorological data). The result is a set of troposphere estimates per station per day which provides internal consistency for the users of the troposphere products currently with a computed standard deviation of the zenith path delay in the range of 1-2 mm (Hackman et al., 2015). The resulting station troposphere files are screened post processing for incomplete files and large standard deviation values to ensure quality troposphere estimate files are passed on to the user community. Below is a summary of the processing parameters (Hackman et al., 2015; Byun and Bar-sever, 2009):

- Fixed orbits and clocks: IGS Final Combined
- Earth orientation: IGS Final Combined
- Transmitter and receiver antenna phase center map: IGS Convention
- Elevation angle cut-off: 7° (receiver dependent)
- Reference frame: International Terrestrial Reference Frame 2008 (ITRF 08)
- A priori troposphere estimate: Global Pressure and Temperature/Niell Model
- Mapping function: GMF (based on data from the global ECMWF numerical model)
- Relative a priori sigma: 1 mm for ZPD and 0.1 mm for each gradient component
• Data window: 27 hours (24 + 3 hours from preceding day to eliminate start-of-day convergence)
• Product interval: 5 minutes
• Estimated parameters: clock (white noise), station position (constant), wet zenith delay (random walk with variance of $12 \text{ mm/} \sqrt{\text{hour}}$), atmospheric gradients (random walk with variance of $0.3 \text{ cm/} \sqrt{\text{hour}}$), phase biases (white noise).

It should be noted that the IGS final products are not suitable for weather forecasting because of their latency of about 12 – 18 days (Johnston et al., 2017).

3.2 Computation of Tropospheric Delays by Ray-tracing through Numerical Weather Models

With the advent of a variety of new meteorological measurements and powerful processing techniques, numerical weather modeling products have matured in recent decades and can produce more accurate weather predictions. The products are also available for larger areas and with higher resolution. The determination of tropospheric delay from NWM is done by ray-tracing. In GNSS analysis, ray-tracing is a procedure used to determine the tropospheric delay of a GNSS signal by assuming that signal to be a ray and tracing that ray along its path, from satellite to receiver through a model for the atmosphere (Nievinski, 2009). The enhanced accuracy and precision of NWMs have made ray-tracing an established technique of determining the total tropospheric
delay, and many researchers worldwide are now evaluating ray-traced delays to correct and model space geodetic observations. On the other hand, for positioning applications that will likely continue estimating residual delays from the observations themselves, NWMs have become the basis for the development of modern mapping functions by way of ray-traced slant delay components.

The geometrical optics approximation is the fundamental principle used in ray-tracing to describe the nature of electromagnetic wave propagation through a slowly varying medium without losing its physical meanings under the assumption of negligible diffraction effects. According to Nievinski and Santos (2010), ray-tracing requires the creation or adoption of an atmospheric model and the two aspects making up a more comprehensive atmospheric model are structure and source. Atmospheric structure defines the arrangement of the surface of constant refractivity or index of refraction, while atmospheric source, on the other hand, denotes the origin of the data making up the atmospheric model (such as radiosondes) or the purpose in generating that atmospheric model (such as weather or climate modeling).
In Figure 3.1 (Nievinski and Santos, 2010), the ray-tracing options are ordered along each axis with the simplest ones closer to the origin and the more rigorous farther away from the origin. The details about these ray-path models and structures, and the comparisons of their combinations in terms of estimating tropospheric delays are outlined in Nievinski (2009). For this thesis, the bent 2-dimensional ray path model in combination with the spherical osculating atmospheric structure was employed, with computed station-specific zenith hydrostatic and wet delays, and horizontal gradients. The ray-tracing was performed at the initialization intervals of 0, 6, 12, and 18 hours (temporal resolution) of the NCEP II global NWM, with a horizontal resolution of 2.5° x 2.5°.
3.3 Least Squares Spectral Analysis

Least Squares Spectral Analysis (LSSA) is based on the developments by Vaniček (1969 and 1971) and later improvements and implementation by Wells et al. (1985) and Pagiatakis (1998) was done to overcome the inherent limitations of the classical Fourier methods. The notable advantages provided by LSSA over Fourier Transforms are:

(i) The analysis of time series with data gaps and unequally spaced values without pre-processing,
(ii) no limitations for the length of the time series,
(iii) time series with an associated covariance matrix can be analyzed,
(iv) the systematic noise can be rigorously suppressed without causing any shift in the existing spectral peaks, and,
(v) statistical tests on the significance of spectral peaks can be performed.

In LSSA, the observed time series \( f \) is considered as a function of time \( t_i \), \( i = 1, 2, \ldots n \). Here, the time series may or may not have equally spaced values. The main objective of LSSA is to determine and clarify the periodic signals in \( f \), especially when \( f \) includes both random and systematic noise. Term \( f \) can be modeled with function \( g \) as follows:

\[
g = Wx,
\]

where \( W \) is a matrix of known base functions and \( x \) is the vector of unknown parameters. The observations \( f_i \) are assumed to possess a fully populated covariance matrix \( C_f \). To estimate the model parameters \( x \), the standard least-squares (LS) method is used, in which the difference between \( g \) and \( f \) is minimized in the least squares sense.
\[
\hat{x} = (W^T C_f^{-1} W)^{-1} W^T C_f^{-1} f, \quad (3.26)
\]

\[
\hat{g} = W\hat{x} = W (W^T C_f^{-1} W)^{-1} W^T C_f^{-1} f. \quad (3.27)
\]

Using the standard least-squares notation, the residual series determined to minimize the difference between \( f \) and \( \hat{g} \) can be written as:

\[
\hat{\nu} = f - \hat{g} = f - W(W^T C_f^{-1} W)^{-1} W^T C_f^{-1} f. \quad (3.28)
\]

Based on projection theorem, it is known that \( \hat{\nu} \) is perpendicular to \( \hat{g} \), meaning that \( f \) has been decomposed into a signal \( \hat{g} \) and noise \( \hat{\nu} \). Thus, a fractional measure \( s \), that describes how \( \hat{g} \) represents \( f \), is given as the ratio of the length of the orthogonal projection of \( \hat{g} \) to the length of \( f \):

\[
s = \frac{f^T C_f^{-1} \hat{g}}{f^T C_f^{-1} f}, \quad (3.29)
\]

where \( s \) is the LS spectrum in percentage \((0 < s < 1)\). The larger the \( s \) (closer to 1) the better is the LS fit to the data. Hidden periodic signals that are expressed in terms of sine and cosine base functions are sought in spectral analysis. Therefore, if a set of spectral frequencies \( \omega_i, i = 1, 2, \ldots m \), are specified, then we can express the signals as:

\[
\hat{g}(\omega_i) = \hat{x}_{1i} \cos \omega_i t + \hat{x}_{2i} \sin \omega_i t. \quad (3.30)
\]

When calculating the LS spectrum, there is a simultaneous least-squares solution for the parameters of the process. This is a rigorous approach to the hidden periodicities: the parameters of the assumed linear system driven by noise are determined
simultaneously with the amplitudes and phases of the periodic components, and with other parameters that describe systematic noise (Pagiatakis, 1998).

From equation (3.30), \( \hat{x} = [\hat{x}_{1i}, \hat{x}_{2i}]^T \) and \( W = [\cos \omega_i t, \sin \omega_i t] \), and \( \hat{x} \) is determined from equation (3.26). For different frequencies \( \omega_i, i =1, 2, \ldots m \), different spectral values can be obtained. The LS spectrum is defined by:

\[
s(\omega_i) = \frac{f^T c_f^{-1} \hat{y}(\omega_i) f^T c_f^{-1}}{f^T c_f^{-1} f} = \left[ 1 + \frac{Q_n}{Q_s} \right]^{-1}, \tag{3.31}
\]

where \( Q_n \) and \( Q_s \) are the quadratic norms of the noise and signal, respectively. Equation (3.31) describes the LS spectrum. The LS spectrum of \( f \) is the collection of the spectral values for all desired frequencies \( \omega_i, i =1, 2, \ldots m \). The greater the spectral value at a frequency \( \omega_i \), the more powerful \( f \) is at this frequency (Erol, 2010).

Statistical testing on the significance of spectral peaks/period components as done by Vaniček (1971) and Pagiatakis (1998), is very important to know which peak is statistically significant and which one can be suppressed. Vaniček (1971) derived the expected (mean) spectral value of white noise in the LS spectrum for series consisting of statistically independent random values. He pointed out the possibility of deriving magnitudes (threshold values) above which spectral peaks are statistically significant.

### 3.3.1 Least Squares Self-Coherency Analysis

The Least Squares Self-Coherency Analysis (LSCA) is based on the LS product theorem, developed by Pagiatakis et al. (2007), which is the product of the LS spectra of the segments of two or more time series under consideration (Mtamakaya, 2012). This
product creates a new probability distribution function that amplifies the common peaks between the time series under study while attenuating the non-common ones. It also establishes rigorous confidence levels for detecting significant peaks.

According to Pagiatakis et al. (2007), the LS product spectrum is a multiplication operation in the frequency domain, and this is equivalent to a time-domain convolution (filtering). When two amplitude spectra corresponding to two time series are multiplied, the resultant cross-amplitude spectrum shows if the amplitude of a component at a specific frequency in one series corresponds with a large or small amplitude at the same frequency in the other series. Also, the cross-phase spectrum informs if a frequency component in one series leads or lags the component at the same frequency of the other series. The cross spectrum comprises of the cross amplitude and phase spectra, both defining the coherency spectrum. A detailed derivation of the LS product spectrum and description of the self-coherency analysis is given in Pagiatakis et al. (2007) and Mtamakaya (2012).
4. Data Processing and Analysis

One of the objectives of this thesis research is determining the ZTD at the NIGNET and IGS stations in and beside the country, as shown in Figure 4.1, from their GNSS observations, using GAPS PPP. Following this is the validation of the PPP-derived ZTD with the corresponding ZTD from the NCEP II global NWM, and from the ZPD from the IGS. Three sets of data (two from GNSS observations and one from NWM) were used to estimate the ZTD and its components. The data sets are described briefly below.

![Figure 4.1 – Map of Nigeria showing NIGNET and IGS stations](image-url)
4.1 GNSS Data Sets – GAPS and IGS

Daily GNSS data for NIGNET and IGS stations spanning January 2011 to December 2016 were downloaded from NIGNET ftp server (server.nignet.net/data/) and IGS ftp server (ftp://cddis.gsfc.nasa.gov/) using the RTKGET application program of RTKLIB (Takasu et al., 2007). Table 4.1 shows the amount of GNSS data downloaded from the NIGNET and IGS servers, with the amount processed from GAPS. Data files not processed by GAPS are because they contain only single frequency observations.

Table 4.1 – Downloaded Data Count (Processed Data Count)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ABUZ</td>
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<td>366 (366)</td>
<td>355 (355)</td>
<td>350 (350)</td>
<td>180 (180)</td>
<td></td>
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<tr>
<td>BKFP</td>
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<td>366 (366)</td>
<td>339 (339)</td>
<td>365 (365)</td>
<td>46 (46)</td>
<td>255 (255)</td>
</tr>
<tr>
<td>CGGT</td>
<td>365 (365)</td>
<td>39 (39)</td>
<td>116 (116)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CLBR</td>
<td>36 (36)</td>
<td>363 (267)</td>
<td>340 (340)</td>
<td>355 (355)</td>
<td>284 (277)</td>
<td>190 (187)</td>
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<tr>
<td>FPNO</td>
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<td>93 (92)</td>
<td>10 (10)</td>
<td>28 (27)</td>
<td></td>
</tr>
<tr>
<td>FUTA</td>
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<td>222 (169)</td>
<td>38 (0)</td>
<td></td>
<td>102 (0)</td>
<td></td>
</tr>
<tr>
<td>FUTY</td>
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<td>361 (361)</td>
<td>289 (287)</td>
<td>290 (290)</td>
<td>168 (168)</td>
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<td>GEMB</td>
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<td></td>
<td>5 (5)</td>
<td>110 (100)</td>
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<tr>
<td>HUKP</td>
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<td>38 (38)</td>
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</tr>
<tr>
<td>KANO</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>256 (256)</td>
<td></td>
</tr>
<tr>
<td>MDGR</td>
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<td>11 (11)</td>
<td>96 (96)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OSGF</td>
<td>299 (299)</td>
<td>363 (363)</td>
<td>335 (335)</td>
<td>94 (94)</td>
<td>156 (156)</td>
<td></td>
</tr>
<tr>
<td>RUST</td>
<td>304 (207)</td>
<td>132 (46)</td>
<td>52 (52)</td>
<td>84 (0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ULAG</td>
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<td>364 (364)</td>
<td>364 (293)</td>
<td>349 (0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNEC</td>
<td>362 (362)</td>
<td>366 (366)</td>
<td>364 (364)</td>
<td>365 (243)</td>
<td>54 (0)</td>
<td>267 (267)</td>
</tr>
<tr>
<td>BJCO</td>
<td>299 (299)</td>
<td>248 (247)</td>
<td>332 (332)</td>
<td>362 (362)</td>
<td>346 (346)</td>
<td>338 (338)</td>
</tr>
<tr>
<td>CGGN</td>
<td>60 (60)</td>
<td>236 (236)</td>
<td>278 (278)</td>
<td>135 (135)</td>
<td>192 (192)</td>
<td>186 (186)</td>
</tr>
</tbody>
</table>
4.1.1 Tropospheric Delay Data from GAPS

The estimation of zenith tropospheric delays and its components by GAPS has been discussed in Chapter 3. Processing with GAPS produces several files, amongst which is a ‘.nad’ file that contains the ZTD information in several columns. In this file, information about the zenith hydrostatic, wet and total delays are provided at every epoch of observation processed (thirty seconds interval). Information about the horizontal North-South and East-West gradients are also provided. Figure 4.2 is a chart of the available ZTD data after processing with GAPS, indicating portions of data gaps (the color variations are meaningless).

Figure 4.2 – Processed ZTD data from GAPS
4.1.2 Tropospheric Delay Data from IGS

The estimation of zenith tropospheric delays and its components by the IGS has been discussed in Chapter 3. The results of the process are compressed in files called zenith path delay (ZPD) files. In these are the information for the ZTD, and the horizontal North-South and East-West gradients, all with their standard deviations, provided at five-minute epochs of observation. RTKGET was also used to obtain the ZPD files from the IGS server. According to Johnston et al. (2017), the IGS ZPD product has an accuracy of 4mm.

4.2 Tropospheric Delay Data from NCEP II Global NWM

These ZTD estimates were obtained through ray-tracing utilizing the meteorological information from NCEP II as discussed in Chapter 3. Since this process of obtaining ZTD is in-house, the file type and the content of the files containing the results can vary. For this thesis, the hydrostatic and wet $a$ coefficients, the zenith hydrostatic and wet delays, and the North-South and East-West horizontal gradients are contained in the file, with temporal resolution of six (6) hours.

4.3 Comparison Between Data Sets

To assess the performance of the PPP-derived ZTD estimates, its comparison between the corresponding NCEP II and IGS estimates (for the IGS stations) was
carried out. The NCEP II and IGS ZTD estimates are the reference estimates in these evaluations due to their high level of precision and because they are considered more established means for the retrieval of ZTD and its components. In PPP static post-processing, the achievement of centimetre-level 3-D coordinates typically occurs within the first few hours of convergence (Abdallah, 2015; Bolbol et al., 2017). For this reason, the initial two hours of the GAPS estimates were not considered in this analysis. Consequently, the zero hour of the NCEP II and the initial two hours of the IGS tropospheric parameters were not considered as well in the analysis.

4.3.1 Comparison Between GAPS and NCEP II

For comparing GAPS with NCEP II, daily 6-, 12- and 18-hour GAPS estimates, averaged over a 5-minute window (2.5 minutes before and after) were used. This averaging was done to minimize the effects of outliers in the six-hourly estimates from GAPS. Figures 4.3, 4.4, 4.5 and 4.6 exemplarily show the GAPS and NCEP II ZTD time series of four stations: BKFP and FUTY in the north, and BJCO and CLBR in the south. The time series for the other stations are given in Appendix B. In general, the GAPS and NCEP II ZTD agree quite well.
Figure 4.3 – ZTD time series for BKFP from NCEP II and GAPS

Figure 4.4 – ZTD time series for FUTY from NCEP II and GAPS
Figure 4.5 – ZTD time series for BJCO from NCEP II and GAPS

Figure 4.6 – ZTD time series for CLBR from NCEP II and GAPS
According to Eludoyin et al. (2014), the two major seasons in Nigeria are the rainy season (April-October) and the dry season (November-March). The quantity of atmospheric water vapor is typically higher in the rainy season, and lower in the dry season and not accounting for the height of the station, higher amounts of atmospheric water vapor are related to higher ZTD estimates and vice versa. Examples of these are seen in both the plots of GAPS and NECP II for the year 2013 as shown in Figures 4.7, 4.8, 4.9 and 4.10. Other years show similar patterns.

Higher ZTD values typically occur within day of the year (DOY) 100 – 300, which coincides with the months of April through October. The lower ZTD estimates, as a result of lower amounts of atmospheric water vapor, are typically found in the dry season months of November through March (around DOY 300 – 365 and 1 – 100). Studies by Olusola et al. (2015) and Willoughby et al. (2002) show that the atmosphere in the south is more humid (more water vapor content) than the northern atmosphere due to the south being closer to the coastline of the Atlantic Ocean.
Figure 4.7 – 2013 NCEP II and GAPS ZTD time series for BKFP

Figure 4.8 – 2013 NCEP II and GAPS ZTD time series for FUTY
Figure 4.9 – 2013 NCEP II and GAPS ZTD time series for BJCO

Figure 4.10 – 2013 NCEP II and GAPS ZTD time series for CLBR
For both the GAPS and NCEP II results, the southern stations have delays generally as high as 2.7 m (Fig. 4.9 and 4.10). However, delays in the northern stations (Fig. 4.7 and 4.8) do not generally exceed 2.65 m. Portrayed within days 200 – 250 of the year (Mid July to early September), is a decrease in GAPS estimates for the BJCO station (Fig. 4.9). This decrease coincides with a phenomenon known as the “August break” (Ogungbenro et al., 2014), which is peculiar to the precipitation pattern in the south-western part of the country and is consistent with the findings of Willoughby et al. (2002). The August break seemingly is not captured by the NECP II estimates as seen in Figure 4.9. Rapid changes in the ZTD estimates from GAPS and NCEP II can be attributed to rapid changes in the humidity around the stations (Lu et al., 2016). Observed gaps at certain epochs in the plots are due to non-availability of GNSS observations from the NIGNET stations.

![Histogram of NCEP II and GAPS ZTD difference for BKFP](image)

Figure 4.11 – Histogram of NCEP II and GAPS ZTD difference for BKFP
Figure 4.12 – Histogram of NCEP II and GAPS ZTD difference for FUTY

Figure 4.13 – Histogram of NCEP II and GAPS ZTD difference for BJCO
Figures 4.11 to 4.14 present the corresponding distributions of ZTD differences between NCEP II and GAPS estimates for stations BKFP, FUTY, BJCO and CLBR for 2011 to 2016. The sign convention is NCEP II – GAPS, with outliers removed. Although the differences appear to be normally distributed, the tau test for the mean of the differences (the sample mean) was carried out to determine if there is statistical evidence that it is significantly different from zero (the population mean). With the null hypothesis being that the sample mean isn’t statistically significantly different from the population mean, the test failed for all stations, indicating that there isn’t statistical evidence that sample mean isn’t statistically different from the population mean. The statistics (the mean differences/biases (μ), standard deviations (σ) and root mean square (rms) values) of the differences between NCEP II and GAPS for all stations between 2011 and 2016 were also determined and are presented in Table 4.2 below. The mean
values of the ZTD differences at the stations range between -6.7 mm to 24.2 mm, with 
RMS ranging between 33.1 mm and 45.7 mm.

Table 4.2 – NCEP II and GAPS ZTD Difference (NIGNET and IGS Stations)

<table>
<thead>
<tr>
<th>Stations (Years of data used)</th>
<th>μ (mm)</th>
<th>σ (mm)</th>
<th>rms (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABUZ (2011-2014, 2016)</td>
<td>3.9</td>
<td>34.7</td>
<td>34.9</td>
</tr>
<tr>
<td>BKPF (2011-2016)</td>
<td>17.0</td>
<td>39.7</td>
<td>43.2</td>
</tr>
<tr>
<td>CGGT (2011-2013)</td>
<td>9.2</td>
<td>37.3</td>
<td>38.4</td>
</tr>
<tr>
<td>CLBR (2011-2016)</td>
<td>-6.7</td>
<td>38.3</td>
<td>38.9</td>
</tr>
<tr>
<td>FPNO (2012-2014, 2016)</td>
<td>16.5</td>
<td>33.7</td>
<td>37.5</td>
</tr>
<tr>
<td>FUTA (2012-2013)</td>
<td>24.2</td>
<td>37.9</td>
<td>44.9</td>
</tr>
<tr>
<td>FUTY (2011-2016)</td>
<td>15.2</td>
<td>41.1</td>
<td>43.8</td>
</tr>
<tr>
<td>GEMB (2012-2013, 2015)</td>
<td>-4.2</td>
<td>32.8</td>
<td>33.1</td>
</tr>
<tr>
<td>HUKP (2012-2015)</td>
<td>10.2</td>
<td>33.9</td>
<td>35.4</td>
</tr>
<tr>
<td>MDGR (2011, 2013-2014)</td>
<td>12.8</td>
<td>40.8</td>
<td>42.8</td>
</tr>
<tr>
<td>OSGF (2011-2014, 2016)</td>
<td>-2.2</td>
<td>39.2</td>
<td>39.3</td>
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<tr>
<td>RUST (2011-2013)</td>
<td>4.4</td>
<td>36.8</td>
<td>37.0</td>
</tr>
<tr>
<td>ULAG (2011-2013)</td>
<td>4.0</td>
<td>43.5</td>
<td>43.6</td>
</tr>
<tr>
<td>UNEC (2011-2014, 2016)</td>
<td>6.9</td>
<td>39.5</td>
<td>40.1</td>
</tr>
<tr>
<td>BJCO (2011-2016)</td>
<td>11.9</td>
<td>44.2</td>
<td>45.7</td>
</tr>
<tr>
<td>CGGN (2011-2016)</td>
<td>13.5</td>
<td>33.0</td>
<td>35.6</td>
</tr>
</tbody>
</table>

Figure 4.15 shows the graph of the mean differences between NCEP II and GAPS ZTD 
estimates of all the stations with their rms above. The stations are arranged according to 
their climatic zones and in order of increasing latitudes. A fitted linear line indicates 
that the rms of the mean differences are generally higher for the southern stations with a 
decreasing trend moving north.
4.3.2 Comparison Between GAPS and IGS

For comparing GAPS and IGS ZTD estimates, the same 5-minute window averaging was done with the GAPS estimates and compared with the IGS five-minute estimates. Figures 4.16 and 4.17 show the GAPS and IGS ZTD time series at every six (6) hours for 2011 to 2016, of BJCO and CGGN respectively. The 6 hours estimates, and not the five-minute estimates, are shown just for clarity purposes, for easier comparison between the GAPS and IGS time series.
Figure 4.16 – ZTD time series for BJCO from IGS and GAPS

Figure 4.17 – ZTD time series for CGGN from IGS and GAPS
In general, the GAPS and IGS ZTD estimates agree much better than between GAPS and NCEP II. An example is the 2013 comparison between GAPS and IGS for BJCO at five-minute intervals as shown in Figure 4.18. Unlike the comparison between GAPS and NCEP II for the BJCO station, both the GAPS and IGS estimates portray the August break in the precipitation pattern of the southwest. The observed gaps at certain epochs in the plots are due to non-availability of observations from the NIGNET station and non-availability of ZPD products from the IGS.

The histogram for the distribution of the ZTD differences between IGS and GAPS estimates for BJCO between 2011 to 2016 is presented in Figure 4.19. The sign convention is IGS – GAPS, with outliers removed, and it can be noticed from the histogram of the differences are normally distributed. The statistics (the mean biases
(μ), standard deviations (σ) and root mean square (rms) values) of the differences between GAPS and IGS for the IGS stations were determined. The precision of the IGS and GAPS comparison agrees well with the result of Guo (2015).

The tau test for the mean of the differences, the sample mean, was done to determine if it is statistically significantly different from the population mean of zero. With the null hypothesis being that the sample mean isn’t significantly different from the population mean, the test failed, indicating that the sample mean is statistically significantly different from the population mean. The statistics of the differences between GAPS and IGS for the IGS stations were determined and are presented in Table 4.3.

Figure 4.19 – Histogram of IGS and GAPS ZTD difference for BJCO
Table 4.3 – IGS and GAPS ZTD Difference (IGS Stations)

<table>
<thead>
<tr>
<th>Stations (Years of data used)</th>
<th>μ (mm)</th>
<th>σ (mm)</th>
<th>rms (mm)</th>
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</thead>
<tbody>
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<td>BJCO (2011-2016)</td>
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<td>6.5</td>
<td>6.8</td>
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<td>CGGN (2015-2016)</td>
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<td>7.7</td>
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4.3.3 Comparison Between GAPS, IGS and NCEP II

For comparing the GAPS, IGS and NCEP II ZTD estimates between one another, only the IGS stations are considered with their comparisons restricted to the epochs which have ZTD estimates from all three sources between 2011 and 2016. Figures 4.20, 4.21 and 4.22 show the histograms of the differences between NCEP II and GAPS, IGS and GAPS, as well as NCEP II and IGS. The histograms of the differences show that they are normally distributed. The histograms of the NCEP II and GAPS and NCEP II and IGS ZTD differences have similar standard deviations and are skewed to the right. The histogram of the IGS and GAPS ZTD differences is skewed left with a much smaller standard deviation indicating better agreement between IGS and GAPS.
Figure 4.20 – Histogram of NCEP II and GAPS ZTD difference for the IGS stations

Figure 4.21 – Histogram of IGS and GAPS ZTD difference for the IGS stations
Figure 4.22 – Histogram of NCEP II and IGS ZTD difference for the IGS stations

Mean = 14.3 mm
Std dev = 42.7 mm
5. Spectral Analysis of ZTD Time Series

ZTD estimated from GNSS is a function of pressure, temperature and water vapor. Because they vary both spatially and temporally, the estimated ZTD behaves likewise. This section presents the LSSA and LSCA of the ZTD estimates from GPS and the NWM, over Nigeria between 2011 and 2016, in order to evaluate their spatial and temporal variabilities. The LSSA version 5.02 program (Vaníček, 1971), which is written in the FORTRAN (Formula Translator) programming language, was used. The distribution of the NIGNET and IGS stations in the different climatic zones of the country is shown in Figure 5.1. These climatic zones and their characteristics have been discussed in chapter 2. The ZTD time series from NCEP II are sampled at six (6) hour intervals. To compare its spectral analysis results to those from the GAPS and IGS estimates, only the exact daily GAPS and IGS ZTD estimates at every 6 hours are used from all stations. Table 4.2 in Chapter 4 lists the amount of daily processed GPS data of the stations for the years 2011 to 2016. Only the stations with a substantial amount of processed data are considered. These stations are ABUZ, BKFP, BJCO, CLBR, CGGN, FUTY, OSGF, ULAG and UNEC.

Computation of the least-square spectrum of the time series requires a data file and an input parameters file. The data file consists of the times of observations serially from the first to the last observations, the ZTD estimates and their standard deviations. The input parameters file known as “lssa.in” contains the user-defined parameters. There are eight blocks of parameters needed to be specified for the analysis of the time series. The definition of these parameters in the lssa.in file, along with a description of
the workings of the LSSA program can be found in the LSSA manual which can be downloaded from http://www2.unb.ca/gge/Research/GRL/LSSA/sourceCode.html.

Figure 5.1 – NIGNET and IGS stations in their Nigerian climatic zones

The first epochs of each of the time series are set as points for datum biases since there are no known shifts in the time series due to changes in the GNSS receivers’ heights. The program was set to determine linear trends in the data, and because the a-priori variance factor is unknown, the weight of each value of the series is calculated as the inverse of the value’s variance. Due to the length of the time series, three bands of 2000 spectral values each were used to capture the periodic signals present in the time
series, with overlaps between the bands. These bands represent the yearly, monthly and daily periods respectively. 2000 is the maximum number of spectral values that can be represented by the program. The first band captures the periods between 4380 hours (approximately half of a year) to the extent of the time series. The second band captures the periods between 432 hours to 5100 hours, and the third band is between 6 hours to 720 hours. The critical level for detecting significant peaks and the level of significance for statistical testing is defined on a 99% confidence level, which represents the most stringent option.

Several runs of the LSSA are carried out, with the strongest period/frequency (that is, the period with the largest significant percentage variance) suppressed in succeeding runs. The percentage variance is the least-squares spectrum. The suppression of strong periods in subsequent runs give rise to new significant periods which may have been weak or invisible in previous runs. Also, from the suppression, the amplitudes and the phases of the strong periods from the preceding runs are given. Only detected significant periods up to half of the length of the time series under analysis are considered. After every run of the LSSA is carried out, several files are created; lssa.out, hist.dat, residual.dat and spectrum.dat. The contents of these files are described in the LSSA manual.

For the LSCA, the product spectra of the ZTD with the ZHD and ZWD are computed. This was done to speculate which of pressure, temperature or relative humidity (moisture content in the troposphere) is the main contributor to the periodic oscillations found in the ZTD by way of using the ZHD and the ZWD. The product
spectra is determined by the summation of the natural logarithms of the percentage variances obtained from the ZTD and ZHD, and the ZTD and ZWD LS spectra. The MATLAB script used for this purpose was adapted from Mtamakaya (2012).

5.1 Spectral Analysis of the ZTD in the Tropical Rainforest Climatic Zone

The stations in this climatic zone are BJCO, CLBR and ULAG. Due to data availability and data gaps within the time series (less than half a year’s worth of ZTD information), observations spanning 2012 to 2016 were considered for CLBR, and 2011 to 2013 for ULAG. Figures 5.2, 5.3 and 5.4 show the GAPS time series plots of the three stations.

![Figure 5.2 – Plot of BJCO GAPS time series](image)

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Figure 5.3 – Plot of CLBR GAPS time series

Figure 5.4 – Plot of ULAG GAPS time series
The spectral plots of the initial runs of the LSSA program for the BJCO and CLBR station are presented in Figures 5.5 and 5.6 respectively, for their GAPS and NCEP II time series. The most dominant peaks across the three bands as seen in the Figures 5.5(i), 5.5(iv), 5.6(i) and 5.5(iv) are those around the annual period (1\textsuperscript{st} harmonic). Also visible are the peaks around the semi-annual (2\textsuperscript{nd} harmonic) and the tri-annual (3\textsuperscript{rd} harmonic) periods in Figures 5.5(ii) and 5.6(ii), and Figures 5.5(v) and 5.6(v) respectively. The spectral plots of the GAPS and NCEP II time series of the ULAG station, shown in Appendix C, also exhibits similar characteristics mentioned above. Considering the percentage variances of the annual peaks in both time series of the three stations in this climatic zone, an increasing trend is observed with increasing latitude. Several other peaks (side lobes) can be seen in the plots but the LSSA does not mark them as significant and the suppression/removal of the significant dominant peaks usually takes away some of the insignificant peaks.
Figure 5.5 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the BJCO station
The plots of the LS product spectra from the LSCA of the BJCO and CLBR stations time series of their ZTD, ZHD and ZWD are given in Figures 5.7, 5.8, 5.9 and 5.10. In these plots, the annual, semi-annual and tri-annual periods can be seen to be amplified, including the daily and sub-daily (high frequency) periods. The plots inform us of the contributions the ZHD and ZWD have to the periodicities seen in the ZTD. The LS product spectra from the LSCA of the ULAG station shows similar results and is included in the Appendix D.
Figure 5.7 – The product spectra of the BJCO ZTD and ZHD GAPS and NCEP II time series
Figure 5.8 – The product spectra of the BJCO ZTD and ZWD GAPS and NCEP II time series
Figure 5.9 – The product spectra of the CLBR ZTD and ZHD GAPS and NCEP II time series
From the plots of the product spectra of the stations in this climatic region, the ZHD is shown to contribute more to the annual oscillations in the GAPS ZTD time series, but not in the NCEP II ZTD time series. However, in both ZTD time series, the ZWD contributes more to the semi-annual oscillations. The ZHD is also seemingly the main contributor to the total of the diurnal and sub-diurnal oscillations in all stations. Table 5.1 presents the computed significant periods and their amplitudes and phases in the three spectral bands considered, that are common in the GAPS, NCEP II and IGS ZTD time...
series of the BJCO station. Tables 5.2 and 5.3 also show the same information but for the CLBR and ULAG stations GAPS and NCEP II ZTD time series respectively.
Table 5.1 – BJCO ZTD Time Series Spectral Result

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Table 5.2 – CLBR ZTD Time Series Spectral Result

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<th>Amplitude (m)</th>
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Table 5.3 – ULAG ZTD Time Series Spectral Result

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<th>Amplitude (m)</th>
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From the tables, the most consistent detected periods for all the stations from both time series are the annual (or close to 365 days), the semi-annual (or around 183 days), and the tri-annual (or around 124 days) in that order from the largest to smaller amplitudes. There are also the 12-hourly (semi-diurnal), the 8-hourly and the 6-hourly periods. For the GAPS times series, the amplitudes of the annual periods increase with
increasing latitude, ranging from 44.9 mm to 48.1 mm. The phases of these amplitudes occur between 176° to 196° which correspond to days in the months of June and July. The BJCO IGS station time series also has its annual period’s amplitude as 46.6 mm and phase at 191.5°. However, this pattern is not evident in the NCEP II time series, with amplitudes ranging from 59.7 mm to 78.6 mm and phases from 198° to 212°. The semi-annual periods for both time series of all stations do not show any obvious latitudinal incremental pattern as shown by the annual periods. For the GAPS and IGS time series, the semi-annual period’s amplitudes range from 24.6 mm to 29.6 mm with similar phases except for the ULAG station. For the NCEP II time series, the semi-annual period’s amplitudes and phases are lower than those of the GAPS time series.

Some other periods consistent with some of the stations from both time series are the quarterly (or around 92 days), forty-seven (47) days and the diurnal periods. Although not shown in the figures or tables above, except for the ULAG station, diurnal periods are detected in the GAPS time series of the other stations. For the NCEP II time series of the stations, there are no detected diurnal periods.

### 5.2 Spectral Analysis of the ZTD in the Tropical Savanna Climatic Zone

The stations in this climatic zone are ABUZ, BKFP and FUTY in the Sudan Savanna, and OSGF and UNEC in the Guinea Savanna. The GAPS time series plots of these stations are shown in Figures 5.11 to 5.15. The ABUZ, BKFP and UNEC stations GAPS time series have significant data gaps in 2015, while that of the FUTY station has
less than half a year’s amount of data in 2016. For this reason, the spectral analysis of the
time series from ABUZ, BKFP and UNEC were done in two parts: without the
significant data gap, from 2011 to 2014, and with the data gap from 2011 to 2016. This
was done to compare their results to see the effects of the data gaps of that magnitude in
the time series. For the GAPS time series of the OSGF station, only 2011 to 2013 was
considered, while 2011 to 2015 was considered for the FUTY station.

![Plot of ABUZ GAPS time series](image)

Figure 5.11 – Plot of ABUZ GAPS time series
Figure 5.12 – Plot of BKFP GAPS time series

Figure 5.13 – Plot of FUTY GAPS time series
Figure 5.14 – Plot of OSGF GAPS time series

Figure 5.15 – Plot of UNEC GAPS time series
The plots of the three bands of the time series spectra from the first runs of the LSSA program for the BKFP and UNEC stations are presented in Figures 5.16 and 5.17. Like the ZTD time series of the Tropical Rainforest stations, the annual periods (signals) have the most dominant peaks with increasing percentage variances as latitude increases. The difference in the peaks portrayed in Figures 5.16(ii) and 5.16(iv) are due to side lobes. These side lobes are removed or suppressed in subsequent runs after the suppression/removal of the significant dominant peaks in preceding runs. The time series spectra of the ABUZ, FUTY and OSGF stations also exhibit these characteristics and are shown in Appendix C.
Figure 5.16 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the BKFP station
Figure 5.17 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the UNEC station

Figures 5.18, 5.19, 5.20 and 5.21 show the plots of the LS product spectra from the LSCA of the BKFP and UNEC stations’ ZTD, ZHD and ZWD time series. In these plots, like the spectra of the stations in the Tropical Rainforest climatic zone, the annual, semi-annual and tri-annual periods can be seen to be amplified, including the daily and sub-daily periods. The plots inform us of the contributions the ZHD and ZWD have to the periodicities seen in the ZTD. The LS product spectra from the LSCA of the ABUZ, FUTY and OSGF stations show similar results and are included in the Appendix D.
Figure 5.18 – The product spectra of the BKFP ZTD and ZHD GAPS and NCEP II time series
Figure 5.19 – The product spectra of the BKFP ZTD and ZWD GAPS and NCEP II time series
Figure 5.20 – The product spectra of the UNEC ZTD and ZHD GAPS and NCEP II time series
From the plots of the product spectra of the stations in this climatic region, the ZWD is shown to contribute more to the annual oscillations in both the GAPS and the NCEP II ZTD time series. However, for the semi-annual oscillations, except for the UNEC station, the ZHD is the main contributor. The ZHD is also seemingly the main contributor to the total of the diurnal and sub-diurnal oscillations in all stations. Tables 5.4 to 5.8 present the significant periods and their amplitudes and phases as computed by the LSSA program in the three spectral bands considered, from the GAPS and NCEP II
ZTD time series of the five stations. For the stations with significant data gaps in 2015, only their 2011 to 2014 results are presented below. The results from their comparisons to those including the gaps are similar.

Table 5.4 – ABUZ ZTD Time Series Spectral Result

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Table 5.5 – BKFP ZTD Time Series Spectral Result

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Table 5.7 – OSGF ZTD Time Series Spectral Result

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</table>
From the tables, and like the spectral results of the time series of the Tropical Rainforest stations, the most consistent detected periods for all the stations from both time series are the annual (or close to 365 days), the semi-annual (or around 183 days), the tri-annual (or around 119 days), the 8 hourly and the 6 hourly periods. Unlike the GAPS time series of stations in the Tropical Rainforest stations however, the amplitudes of the tri-annual periods in the time series of some of the stations in the Tropical Sudan Savanna sub-climate (ABUZ and BKFP) are higher than those of their semi-annual periods. The pattern of increasing amplitudes of the annual periods with increasing latitude is also seen in both time series, ranging from 66.8 mm to 116.7 mm for the GAPS time series, and 72.3 mm to 114.6 mm for the NCEP II time series. The phases of the annual periods of the GAPS time series range from 202° to 207° for the stations in the Tropical Sudan Savanna sub-climate, and 184° to 186° for the stations in the Tropical Guinea Savanna sub-climate. For the NCEP II time series, the phases of the annual periods for both sub-climates range from 190° to 198°. The semi-annual periods for both time series of all stations show latitudinal decrease contrary to what is shown by the annual periods. Other similar periods seen in both time series of the stations of the Tropical Guinea Savanna sub-climate are around 92 days (quarterly) and 47 days. For the time series of the Tropical Sudan Savanna stations, periods around 73 to 78 days and around 47 days are also generally seen. No significant diurnal periods are detected in
both their GAPS and NCEP II time series. Except for the UNEC station’s GAPS time series, all other stations in both time series have semi-diurnal periods, although with no discernable pattern across the latitudes.

5.3 Spectral Analysis of the ZTD in the Montane Climatic Zone

There’s only one station in this climatic zone, CGGN. The GAPS time series plot of this station is shown in Figure 5.17.

![Figure 5.22 – Plot of CGGN GAPS time series](image)

The time series of the CGGN station has significant data gaps in 2011 so only 2012 to 2016 was considered. The plots of the three bands of the GAPS and NCEP II time series spectra from the first runs of the LSSA program for CGGN are presented in
Figure 5.23. As with the ZTD time series of the Tropical Rainforest and Savanna stations as seen in the plots of the first band, the annual periods (signals) are the most dominant peaks with percentage variances as high as those in the Tropical Savanna stations. In the plots of the second band, the GAPS time series shows a dominant period just more than 4 months, and the NCEP II time series shows a dominant period just less than 6 months instead.

Figure 5.23 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the CGGN station
Figures 5.24 and 5.25 show the plots of the LS product spectra from the LSCA of the CGGN station’s ZTD, ZHD and ZWD time series. Like the spectral plots of the time series of the stations in the Tropical Rainforest and Savanna climatic zones, the annual, semi-annual and tri-annual periods can be seen to be amplified, including the daily and sub-daily periods. The contributions that the ZHD and ZWD have to the periodicities seen in the ZTD can be deduced from the plots.

Figure 5.24 – The product spectra of the CGGN ZTD and ZHD GAPS and NCEP II time series
From the plots of the product spectra of the CGGN station, the ZWD is shown to contribute more to the annual oscillations in both the GAPS and the NCEP II ZTD time series. However, for the semi-annual oscillations, the ZHD is the main contributor. It is not obvious if the ZHD or ZWD contributes more to the total of the diurnal and sub-diurnal oscillations in both ZTD time series. Table 5.9 presents the strongest and/or recognizable significant periods and their amplitudes and phases as computed by the
LSSA program in the three spectral bands considered, from the GAPS and NCEP II ZTD time series of station CGGN.

Table 5.9 – CGGN ZTD Time Series Spectral Result

<table>
<thead>
<tr>
<th></th>
<th>GAPS</th>
<th>NCEP II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period (days)</td>
<td>Amplitude (m)</td>
<td>Phase (degrees)</td>
</tr>
<tr>
<td>364.2061</td>
<td>0.1080</td>
<td>201.4119</td>
</tr>
<tr>
<td>194.9661</td>
<td>0.0111</td>
<td>286.8747</td>
</tr>
<tr>
<td>75.7460</td>
<td>0.0155</td>
<td>49.9455</td>
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<tr>
<td>49.8252</td>
<td>0.0083</td>
<td>135.3823</td>
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<tr>
<td>31.7033</td>
<td>0.0090</td>
<td>258.5727</td>
</tr>
<tr>
<td>18.8053</td>
<td>0.0045</td>
<td>194.102</td>
</tr>
<tr>
<td>0.5001</td>
<td>0.0036</td>
<td>250.9868</td>
</tr>
<tr>
<td>0.2545</td>
<td>0.0080</td>
<td>143.5247</td>
</tr>
</tbody>
</table>

From the table, the most consistent detected periods from both time series are the annual (or close to 365 days), the semi-diurnal (12 hours) and the 6-hourly. The amplitude and phase of the annual period of the GAPS time series, given as 108 mm and 201.4° respectively, are comparable with those of the stations in the Tropical Sudan Savanna sub-climate. With an amplitude and phase of 122.9 mm and 187.5° respectively, the annual period of the NCEP II time series are comparable with those of the Tropical Sudan Savanna sub-climate stations. 194 and 178 days, from the GAPS and NCEP II time series respectively, are the closest to the semi-annual periods seen in the time series of other stations.
6. Conclusions and Recommendations

6.1 Summary

In Global Navigation Satellite System (GNSS) positioning, the zenith tropospheric delay (ZTD) delay is determined and eliminated to obtain accurate point positions, but in atmospheric studies and research such as meteorology, this delay is very useful. This dissertation deals with the assessment of the ZTD estimated from the precise point positioning (PPP) technique of Global Positioning System (GPS) observations of the continuously operating reference stations (CORS) in the Nigerian GNSS Reference Network (NIGNET) and from the International GNSS Service (IGS). NIGNET was primarily established as a platform for the use of modern space geodetic techniques for surveying, mapping, land administration and management and high precision geodesy. As is done by many agencies, organizations and institutions involved in meteorological science across the globe, information about ZTD is assimilated into numerical weather models (NWM) for better short-term weather forecasting and long-term climate studies.

The PPP program used for this purpose is the GNSS Analysis and Positioning Software (GAPS). The quality of the GAPS PPP-derived ZTDs from GPS observations is assessed by comparisons to the ZTDs computed from a global NWM and those obtained from the IGS. The National Centers for Environmental Prediction Reanalysis-2 (NCEP II) is the global NWM used in this work. Time series spectral analyses of the ZTDs from the three (3) sources was also performed using Least Squares Spectral Analysis (LSSA) and Least Squares Self-Coherent Analysis (LSCA). The inherent short-term periodic
oscillations that drive the temporal and spatial trends found in the time series of the ZTDs from the 3 sources were compared.

6.1.1 Assessment of the Comparison between the ZTD Estimates from GAPS, NCEP II and IGS

The plots of the ZTD estimates from the GAPS, NCEP II and IGS for the various GNSS stations around the country generally corroborate, temporally and spatially, with the rainy (wet) and dry weather patterns across the country. After the removal of outliers, the comparison between the GAPS and NCEP II estimates, on a station by station basis, has mean differences ranging from -6.7 mm to 24.2 mm with standard deviations ranging from 32.8 mm to 44.2 mm. While there are no observable spatial trends in the mean differences across the stations, the standard deviations and root mean square errors of the differences for stations in the Tropical Rainforest climatic region are generally higher than those in the other climatic regions. This may be attributed to the variability and the higher concentration of atmospheric water vapor typical of coastal equatorial regions.

The comparisons of the estimates on a source basis involved only the IGS stations. The GAPS and IGS estimates have a mean difference of -2.1 mm and a standard deviation of 6.7 mm. The GAPS and IGS estimates have similar comparisons with those from the NCEP II with mean differences of 12.4 mm and 14.3 mm and standard deviations of 42.8 mm and 42.7 mm respectively. The comparisons between the NCEP II and the GAPS and IGS estimates is weaker than the comparison between the GAPS and IGS estimates. The stronger agreement between GAPS and IGS can be attributed to both being obtained through the same technique, albeit with differences in the elevation cut-
off angles (10° for GAPS and 7° for IGS) and a priori tropospheric models and mapping functions (VMF1 for GAPS and Niell model and Global Mapping Functions for IGS) used. The weaker comparisons with the NCEP II estimates could be due to the lack of information from the NIGNET stations being assimilated into the NCEP II NWM. This would prevent the NWM from sufficiently representing the atmospheric conditions in the country. The weaker comparisons with the NCEP II estimates may also be due to the resolution of the NCEP II NWM.

6.1.2 Assessment of the Time Series Analysis of the ZTD Estimates from GAPS, NCEP II and IGS

Based on the results in Chapter 5, it can be concluded that the ZTD time series from the three ZTD sources of most of the stations contain strong periodic oscillations/signals of first, second and third harmonics in a year. They also contain higher frequency sub-daily oscillations. The annual (first harmonic) signals in the spectra of the time series have the highest amplitudes with their phases occurring around the peak of the rainy months in Nigeria. The amplitudes of the annual signals show an increasing trend from stations in the Tropical Rainforest climatic region to stations in the Tropical Savanna climatic region. This corroborates with the results of Jin et al. (2007) where the amplitude of the annual period oscillation increases with increasing latitude from the equator up to mid-latitude areas. According to Isioye et al. (2017), ZTD follows a strong seasonal cycle with it being at its highest in the rainy season and at its lowest in the dry season. Also, the seasonal cycle in the southern area of the country is not as strong compared to the that in the north. This seasonality is influenced by the saturation vapor
pressure set by local temperatures but is also modulated by the seasonal influence of the air masses.

Although the annual signals have the highest amplitudes in all stations, the order of the amplitudes of the semi-annual (2\textsuperscript{nd} harmonic) and tri-annual (3\textsuperscript{rd} harmonic) signals are different across the climatic zones. In the Tropical Rainforest and Tropical Guinea Savanna climatic regions, the amplitudes of the semi-annual signals in the spectra of both the GAPS and NCEP II ZTD time series of the stations are higher than those of the tri-annual signals. The situation is the opposite moving up in latitude to the Tropical Sudan Savanna climatic region. Across all stations, there are no noticeable increasing or decreasing latitudinal trend for the amplitudes of the tri-annual and sub-daily signals. There are other detected significant oscillations present in the time series of all stations. Some of these oscillations seem to be local, for example, the approximate quarterly (4\textsuperscript{th} harmonic) and 47-day oscillations which are detected in the time series of the BJCO and CLBR stations in the Tropical Rainforest climatic zone, and in the UNEC station of the Tropical Savanna climatic zone. However, some other detected oscillations cannot be clearly explained.

Since the ZHD and ZWD are modelled from meteorological parameters such as pressure, temperature and relative humidity, the contributions from these parameters to the periodic oscillations in the ZTD can be evaluated. To achieve this, LSCA was performed by computing the product spectra of the time series LS spectra of the ZHD and ZWD with that of the ZTD. This was computed separately for the GAPS and NCEP II time series to compare their resulting harmonic signatures. The results show periodic contributions from both the ZHD and ZWD to the ZTD. However, the ZWD has the
higher contribution to the strongest periodic oscillations (the annual signal) in both the GAPS and the NCEP II ZTD time series of all stations, except for the GAPS times series of the stations in the Tropical Rainforest climatic region. The ZWD also contributes more to the semi-annual periodic oscillations in the time series of the stations in the Tropical Rainforest climatic region.

The atmosphere in the northern part of Nigeria has been observed to have the largest temperature variations in the country (Okoh et al., 2017) as well as the highest moisture content change by way of relative humidity change from the dry to rainy season. The variable nature of the ZTD is caused largely by the ZWD. With the use of relative humidity and temperature in the estimation of the ZWD, it can be seen how the ZWD would be the main contributor to the strongest periodic oscillations in the ZTD time series, especially of stations in the north. The higher contribution of the ZWD to the strongest periodic oscillations found in the ZTD time series is also confirmed by Jin et al. (2007). The southern part of the country is known to have the least relative humidity and temperature variations, which could be why there’s lower contribution from the ZWD to the annual periodicity in the ZTD time series of stations in the Tropical Rainforest climatic zone. However, for the GAPS time series of the stations in this climatic zone, there’s more contribution from the ZHD than from the ZWD. The reason for this is unknown. For the diurnal and sub-diurnal oscillations in the ZTD times series, the ZHD was found to be the main contributor from the results. With the ZHD composed of pressure and temperature, the results agree with Klos et al. (2016) that the diurnal variations arise from changes in the local pressure and temperature. Also, the daily

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periodic heating of the atmosphere by the sun creates solar atmospheric tides that have the diurnal and sub-diurnal periods found in the ZTD time series of all the stations.

6.2 Recommendations

This study has assessed the quality of the PPP estimated ZTD from the GPS observations from the various NIGNET stations through comparisons with the ZTD computed from the NCEP II NWM and ZTD obtained from the IGS. All over the globe, networks of GNSS stations are utilized as observational methods for studying the atmosphere (GNSS meteorology), with their ZTD data assimilated into various NWMs. In Nigeria, NIGNET can be used as an additional observational method for meteorology, filling the existing gaps of the current observational systems, based on its low cost, all-weather measuring capability and good temporal resolution. The utilization of NIGNET to explore the Nigerian weather and climate adequately requires further efforts. It is suggested that the NIGNET should be densified and properly maintained and managed. Studies have shown that such assimilation is valuable as it improves the performance of NWMs, as well as its forecasting capabilities.

Another valuable product from GNSS observations that is assimilated into NWM is the precipitable water vapor (PWV) which can be obtained to a great degree of accuracy from the ZWD and surface meteorological data (temperature) from a collocated meteorological station. Having collocated meteorological sensors with GNSS stations not only helps with the accurate estimation of PWV but also in the evaluation of the ZTDs for their accuracy. Currently, no known study has reported on the assimilation of ZTDs
and PWVs from Nigeria into any global or region NWM, and so information from these NWMs may be expected not to accurately/sufficiently represent the state of the atmosphere in Nigeria. Undoubtedly, the ZTDs and PWVs from a dense NIGNET assimilated into various NWMs will address this issue, as such assimilation could improve weather predictability. This is particularly useful for determining adverse effects like flooding or drought from severe weather phenomena and taking appropriate actions to reduce the likely damage they can cause.

With a collection of ZTD and PWV time series, which would be expected to be consistent from a densified and properly managed NIGNET, better time series analysis can be carried out. For example, in this report, not much emphasis was placed on the interpretation of the phases of the oscillations, primarily due to the inconsistency (data gaps) in the time series. In future studies, with very consistent time series, an analysis with emphasis on the phases of the detected periodic oscillations could be done. This analysis would facilitate a better understanding of the characteristics of these phases in the various climatic zones and how they relate to the occurrence known weather phenomena in the country. Discussions about climate change in the country was not addressed in this report due to the short period of consideration in the study (2011–2016) compared to the generally accepted period of observable changes due to climate change (over 30 years). A long-term consistent time series is expected to produce well-defined periodic oscillations with amplitude and phases which would be immensely beneficial to climate research in the country.
Bibliography


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Appendix A: Selected GAPS Processing Options

The table below lists the GAPS processing options used in this thesis.

Table A. 1 – Selected GAPS Processing Options

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<th>Item</th>
<th>Options Used</th>
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<td>Processing mode</td>
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<td>Elevation cut-off</td>
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<td>Tropospheric delay</td>
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<tr>
<td>Ambiguities</td>
<td>Estimated as real numbers</td>
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<tr>
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<td>Solid Earth tide, ocean tide loading IERS Convention 2010</td>
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<tr>
<td>Satellite antenna</td>
<td>Phase Centre Offset (PCO) &amp; Phase Centre Variation (PCV) corrected</td>
</tr>
<tr>
<td>Receiver antenna</td>
<td>PCO &amp; PCV corrected</td>
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<tr>
<td>Orbit and Clock products</td>
<td>IGS final product</td>
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Appendix B: GAPS and NCEP II Time Series of ABUZ, CGGN, OSGF, UNEC and ULAG

Figure B. 1 – ZTD time series for ABUZ from NCEP II and GAPS
Figure B. 2 – ZTD time series for CGGN from NCEP II and GAPS

Figure B. 3 – ZTD time series for OSGF from NCEP II and GAPS
Figure B. 4 – ZTD time series for UNEC from NCEP II and GAPS

Figure B. 5 – ZTD time series for ULAG from NCEP II and GAPS
Appendix C: Time Series Spectra of ULAG, ABUZ, FUTY and OSGF

Figure C. 1 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the ULAG station
Figure C. 2 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the ABUZ station
Figure C. 3 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the FUTY station
Figure C. 4 – The 3 bands of the 1st run of the GAPS and NCEP II ZTD time series spectra of the OSGF station
Appendix D: Product Spectra of the ULAG, ABUZ, FUTY and OSGF ZTD, ZHD and ZWD Time Series

Figure D. 1 – The product spectra of the ULAG ZTD and ZHD GAPS and NCEP II time series
Figure D. 2 – The product spectra of the ULAG ZTD and ZWD GAPS and NCEP II time series
Figure D. 3 – The product spectra of the ABUZ ZTD and ZHD GAPS and NCEP II time series
Figure D. 4 – The product spectra of the ABUZ ZTD and ZWD GAPS and NCEP II time series
Figure D. 5 – The product spectra of the FUTY ZTD and ZHD GAPS and NCEP II time series
Figure D. 6 – The product spectra of the FUTY ZTD and ZWD GAPS and NCEP II time series
Figure D. 7 – The product spectra of the OSGF ZTD and ZHD GAPS and NCEP II time series
Figure D. 8 – The product spectra of the OSGF ZTD and ZWD GAPS and NCEP II time series
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